

# Hot-carrier fluctuations from ballistic to diffusive regime in submicron semiconductor structures

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**Abstract.** We present a theoretical analysis of number and current fluctuations in homogeneous n-Si resistors of submicron dimensions at increasing electric field strengths. To this purpose, we calculate the corresponding correlation functions. In the ballistic regime a simple scaling relationship of the transit time accounting for carrier heating is found to hold over the whole range of fields considered. In the diffusive regime different time scales associated with diffusion, drift and dielectric relaxation are found to characterize the behaviour of number fluctuations.

## 1. Introduction

The recent trend in scaling down the dimensions of a device deep into the submicron region has emphasized the importance of a better understanding of fluctuations, since their relative value in ensemble averages is in general proportional to the inverse of the carrier number (i.e. fewer carriers result in more noise). Furthermore, since in small structures the electric field may reach quite large values, the onset of hot-carrier conditions [1] strongly influences the characteristics and the level of fluctuations with respect to the simpler Ohmic condition. In a large system fluctuations are governed by scattering processes, and the associated correlation functions, as a rule, exhibit a standard exponential shape. However, as the size of the system decreases, scattering processes lose their importance and the role of the contacts becomes dominant [2]. The correlation functions are thus expected to exhibit substantial deviations from the standard shape.

The aim of this paper is to present analytical and Monte Carlo calculations of the number and current fluctuations in submicron semiconductor structures. As a matter of fact, the shape of the associated correlation functions can be directly related to the characteristic times of electronic transport, in particular the finite transit time of carriers through the device. Therefore, a

study of the full time dependence of the correlation functions in submicron structures can provide additional insight into the microscopic dynamics in the absence as well as in the presence of dissipation.

## 2. Theory

Fluctuations of a given stochastic variable  $A(t)$  are analysed here in terms of its correlation function  $C_A(t) = \overline{\delta A(0)\delta A(t)}$ , where  $\delta A(t) = A(t) - \bar{A}$  is the fluctuation of  $A$  around the mean value  $\bar{A}$ , and the bar indicates time average (ergodicity is assumed to hold throughout the present model). By taking conditions of constant-voltage operation, the stochastic variables that will be studied are the total number of carriers present instantaneously in a two-terminal sample,  $N(t)$ , and the total current measured in the external circuit,  $I(t)$ .

We consider a one-dimensional resistor of length  $L$  embedded at the centre of a longer resistor which acts as contact and carrier reservoir. The length  $L$  can be shorter (longer) than the carrier mean free path so that ballistic (diffusive) regimes can be analysed. In all cases the carrier concentration is assumed to be sufficiently small to neglect degeneracy effects. Calculations are applied to the case of n-Si at 300 K using the same microscopic model as in reference [3].

At equilibrium an analytical expression for the correlation functions can be obtained in the ballistic case

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according to [4, 5]

$$C_N(t) = \bar{N} \left\{ \operatorname{erf}\left(\frac{\tau_T}{t}\right) - \frac{t}{\sqrt{\pi}\tau_T} \left[ 1 - \exp\left(-\frac{\tau_T^2}{t^2}\right) \right] \right\} \quad (1)$$

$$C_I(t) = \frac{\bar{N}e^2}{2\tau_T^2} \left\{ \operatorname{erf}\left(\frac{\tau_T}{t}\right) - \frac{2t}{\sqrt{\pi}\tau_T} \left[ 1 - \exp\left(-\frac{\tau_T^2}{t^2}\right) \right] \right\} \quad (2)$$

where  $\tau_T = L\sqrt{m/2K_B T}$  is the average transit time,  $e$  being the absolute value of the electronic charge,  $m$  a scalar effective mass,  $K_B$  the Boltzmann constant,  $T$  the lattice temperature, and  $\operatorname{erf}$  denotes the error function. In the diffusive case, while  $C_I(t)$  is known to decay exponentially on the time scale of momentum relaxation, a decay on a characteristic transit-time scale can be expected for  $C_N(t)$  since the number of carriers can change only due to injection or absorption by the contacts.

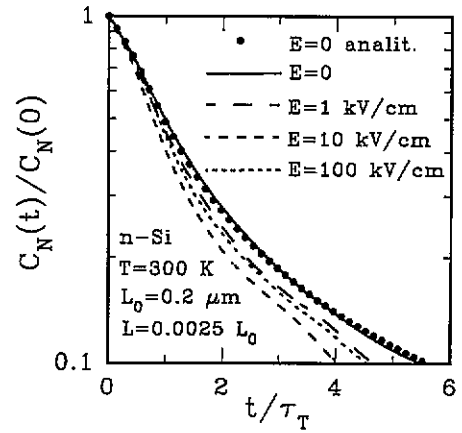
Under non-equilibrium conditions, in general, no analytical expressions for the correlation functions are available. Therefore, we can rely only on numerical solutions of the appropriate kinetic equation, which will be carried out within an ensemble Monte Carlo simulator coupled with a one-dimensional Poisson solver.

Let us now discuss the time scales that we expect to be relevant for  $C_N(t)$ . Plausible candidates, in the presence of an electric field  $E$ , are: the diffusion time  $\tau_D = L^2/D(E)$ , the drift time  $\tau_{vd} = L/v_d(E)$ , and the differential dielectric relaxation time  $\tau_e = \epsilon/en\mu'(E)$ . Here  $D$  is the longitudinal diffusion coefficient,  $v_d$  the average drift velocity,  $\epsilon$  the dielectric constant of the material,  $n$  the carrier concentration and  $\mu'$  the differential mobility. Because the decay of correlations should be limited by the shortest of these times, we introduce the characteristic time  $\tau$ , the reciprocal of which is given by summing the reciprocals of the above three times, according to

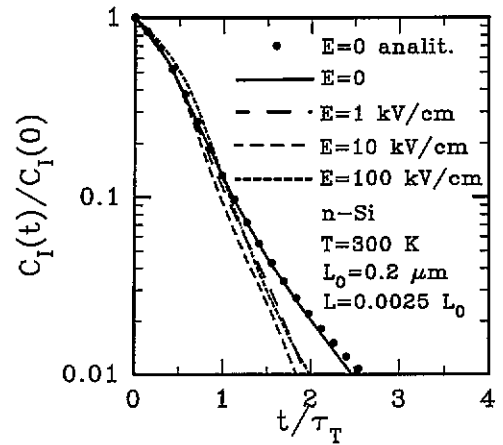
$$\tau = (\tau_D^{-1} + \tau_{vd}^{-1} + \tau_e^{-1})^{-1}. \quad (3)$$

### 3. Results and conclusions

The correlation functions are calculated by considering the transport properties of a small slice of length  $L$  of a homogeneous resistor of length  $L_0$  with local-periodic boundary conditions. To this end, the device is divided into meshes of equal length (about 100). Then we keep the first and last meshes neutral at all times, thus realizing a good model for an Ohmic contact. When the length of the sample becomes much smaller than the carrier mean free path, the particles fly ballistically between the contacts and the results can be compared with those of the analytical calculations. These results are shown in figures 1 and 2, where, together with the Monte Carlo results (curves), we have reported the analytical values (symbols) obtained from equations (1) and (2). The excellent agreement between the analytical and the Monte Carlo results at equilibrium validates the numerical approach used. We notice that both  $C_N$  and  $C_I$  exhibit a non-exponential behavior; in particular, in the limit of  $t \rightarrow \infty$  they are proportional to  $t^{-1}$  and  $t^{-3}$  respectively. The long-time tail of  $C_N$  (which leads to a



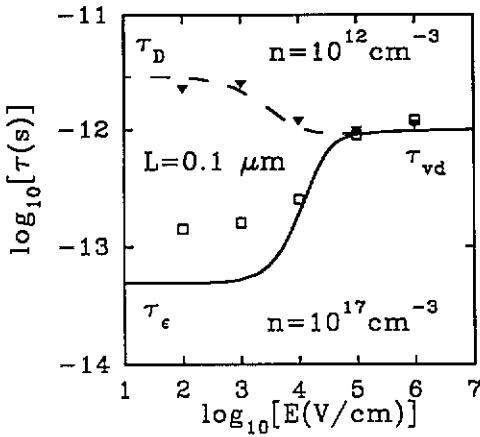
**Figure 1.** Normalized autocorrelation functions of number fluctuations for an n-Si resistor in the ballistic regime for the reported electric field strengths and geometrical parameters. The symbols are obtained from equation (1) and the curves report the results of Monte Carlo calculations.



**Figure 2.** Normalized autocorrelation functions of current fluctuations for an n-Si resistor in the ballistic regime for the reported electric field strengths and geometrical parameters. The symbols are obtained from equation (2) and the curves report the results of Monte Carlo calculations.

logarithmic divergence in the associated low-frequency spectral density) is related to carriers injected with low velocity, which are always present in a Maxwell-Boltzmann distribution. Hence, in the absence of scatterings all carriers emitted by one contact reach the opposite one. Since the transit time of slow carriers tends to infinity, the long-time correlation function decays as  $t^{-1}$  (see equation (1)). The above long-time tail is smoothed in  $C_I$ , since for the calculation of the current the carriers are weighted by their velocity. As a consequence, slow carriers give a smaller contribution to the correlation function, and  $C_I$  decays to zero more rapidly than  $C_N$  by finally exhibiting an asymptotic  $t^{-3}$  dependence.

On the basis of the above results, we have extended our analysis to non-equilibrium conditions by applying a voltage at the boundaries of the resistor, so that hot-carrier conditions set in. In this situation, the

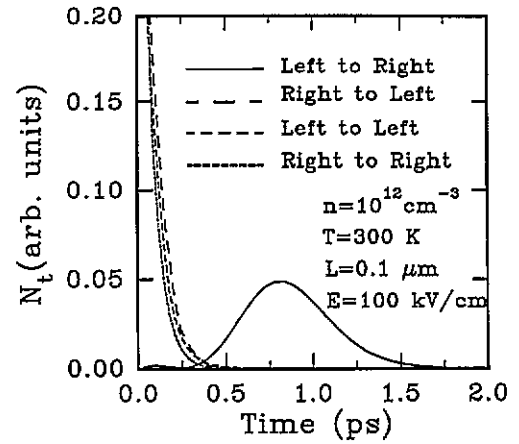


**Figure 3.** Characteristic times associated with number fluctuations as a function of the electric field in the diffusive regime for the reported parameters. The broken curve and the full triangles refer to calculations for a low carrier concentration of  $10^{12} \text{ cm}^{-3}$ ; the full curve and the open squares refer to calculations for a high carrier concentration of  $10^{17} \text{ cm}^{-3}$ .

remaining resistor, which represents the contacts in the Monte Carlo simulation, injects carriers according to a far-from-equilibrium distribution function. The Monte Carlo results show that if the transit time  $\tau_T$  defined above is replaced by  $L\sqrt{m/2K_B T_e}$  where  $T_e = 2\langle\epsilon\rangle/3K_B$ ,  $\langle\epsilon\rangle$  being the average carrier energy, a good agreement is found with the analytical formula over the whole range of voltages considered here. Indeed, with respect to the equilibrium conditions, the presence of an applied electric field is found to be responsible for a faster decay at long times of both the correlation functions.

In the diffusive regime we have investigated the number fluctuations at increasing field strengths. Even in this case  $C_N(t)$  is found to exhibit a non-exponential shape [6]. Anyway, from the decay of the correlation function we have extracted a characteristic time, which has been compared with that conjectured in equation (3). This latter has been determined by using phenomenological expressions for  $D$ ,  $v_d$  and  $\mu'$  as functions of  $E$  [7]. In figure 3 we have reported the results obtained from the Monte Carlo and the analytical calculations for two different carrier concentrations: a low one equal to  $10^{12} \text{ cm}^{-3}$  and a high one equal to  $10^{17} \text{ cm}^{-3}$ , where dielectric relaxation becomes important while degeneracy can still be neglected at 300 K. The good agreement found between the Monte Carlo results and the analytical expression for  $\tau$  supports our conjecture. Furthermore, we remark that, at low density, the characteristic time goes from the long value of  $\tau_D$  to the short one of  $\tau_{vd}$ . In contrast, at high density, it goes from the short value of  $\tau_e$  to the long one of  $\tau_{vd}$ . Here, the slight discrepancy between analytical and Monte Carlo results is due mostly to the presence of plasma effects.

To better clarify the characteristic times of the number fluctuations at high electric fields, we have reported in figure 4 the histograms of the times spent by the carriers inside the device, in order to obtain direct information on the transit times of the carriers inside the sample.



**Figure 4.** Histograms of the number of times  $N_t$  spent by the carriers inside an n-Si resistor in the diffusive regime for the reported parameters. Carriers are drifting from the left to the right contact.

From this figure it can be seen that the different time histograms exhibit structures in two different time regions. The first one (at short times) shows an exponential decay and is connected with the carriers which, due to scattering processes, return to the contact from which they originated, thus spending a very short time in the sample. The second one (at longer times) peaks at the average transit time of carriers going from one contact to the other, which in this case is given by  $\tau_{vd}$ . Since the presence of the scattering processes does not change the actual number of carriers inside the device, this result shows that the situation is similar to that found for the current fluctuations of a classical injector with constant velocity in a ballistic sample [5].

In conclusion, we have presented a detailed analysis of number and current fluctuations in submicron semiconductor structures in the ballistic and diffusive regimes. The main results of the calculations are summarized as follows. Under ballistic conditions: (i) at equilibrium we have found an excellent agreement between the analytical expressions for the autocorrelation functions and the corresponding Monte Carlo calculations. Furthermore, evidence has been found for a long-time tail related to the dominant role played by carriers with small velocity. (ii) For the case of hot-carrier conditions a simple scaling relationship of the transit time is found to hold over the whole range of fields considered. In the diffusive regime, different time scales connected with diffusion, drift and dielectric relaxation are shown to characterize the behaviour of number fluctuations. In all cases considered here, the correlation function of number fluctuations exhibits a non-exponential behaviour with a shape that reflects the complexity of the mechanisms responsible for the transit time of carriers through the sample.

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