TRANSFER-FIELD METHODS FOR ELECTRONIC NOISE IN SUBMICRON SEMICONDUCTOR STRUCTURES

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List of acronymous

AF Admittance Field

BL Boltzmann Langevin

BTE Boltzmann Transport Equation

DD Drift and Diffusion

HD Hydrodynamics

IF Impedance Field

MC Monte Carlo

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Abstract

We review the impedance field (IF) method originally proposed by Shockley, Copeland and James in 1966 and further developed up to the present time by focusing on its implementation for the calculation of electronic noise in deep submicron semiconductor devices. The review mainly consists of two parts. The first part makes use of the standard velocity fluctuation scheme, which relates voltage fluctuations ultimately to single carrier velocity fluctuations. At the microscopic level the relevant system of equations for the IF method is derived from the Boltzmann-Langevin equation. It is shown that the spectral strength of the diffusion noise source entering the IF method is related to the part of conduction current fluctuations caused only by velocity fluctuations in the presence of a static electric field profile. Accordingly, an extended IF relation including two-point cross-correlations of conduction-current fluctuations is derived using the Green-function formalism. The analytical theory is validated by comparing the results obtained with a mixed hydrodynamic-Monte Carlo modeling with those calculated with a full Monte Carlo simulation in a submicron n^+nn^+ GaAs structure. In the mixed modeling the hydrodynamic approach is used to calculate the IF, and the Monte Carlo simulation is used to evaluate the spatio-temporal correlation of velocity fluctuations in the presence of a static electric field profile. Because of the violation of spatio-temporal symmetry due to non-homogeneous structures and the presence of a current flux, the spectral strength of the diffusion-noise source is shown to be a complex quantity associated with conduction current fluctuations caused only by velocity fluctuations. The main drawbacks of the traditional IF method and extensions are overcome by the technique reported in the second part of the paper, which is devoted to the development of a hydrodynamic-Langevin approach in the framework of hydrodynamic equa-

tions which include the Langevin forces. In this approach the microscopic noise sources are related to the instantaneous random accelerations which are originated by single scattering events and are uncorrelated both in time and space. Such a scheme allows us to describe all transfer phenomena related to the spatio-temporal evolution of fluctuations inside the structure in the framework of a unified formalism based on the construction of linear response functions to a single local perturbation caused by a scattering event, i.e. in terms of the Green functions. The spectral power of equivalent Norton or Thevenin generators is represented now in the same way by using the generalized impedance and admittance fields. These generalized transfer fields relate the local power of noise sources inside the structure with the power of fluctuations induced by them at the structure terminals. The spatio-temporal evolution of fluctuations, its influence on the transfer fields and the noise are analyzed in the framework of a simple analytical model of an inhomogeneous structure and illustrated by numerical calculations for submicron GaAs n^+nn^+, n^+n-Schottky-contact and Si p^+n structures.

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I. INTRODUCTION

With the discovery of the transistor [1] and the explosive application of semiconductor materials in many fields of strategic importance (computing, telecommunications, electronics in general, etc.) noise characterization and its microscopic interpretation remained crucial up to present days. The recent trend of scaling down the dimensions of a device below the micron size has further emphasized the importance of a better understanding of far-from-equilibrium phenomena. This is the case of modern semiconductor structures, where hot carrier effects due to local electric fields of very strong intensity (above 10 kV/cm) modify essentially both the small signal response (admittance or impedance) and the electronic noise. Furthermore, the presence of space charge due to contact injection and/or gradients of carrier concentration inside the structures also modifies essentially the noise spectra of the whole structure by coupling current and electric field fluctuations both in space and time. Even from an applied point of view, the knowledge of the noise spectrum for a given sample or device gives objective criteria in looking for the limits of its performances and serves in many instances as a diagnostic tool for the fabrication technology [2]. However, due to the complex geometry of these modern devices and the large variety of physical effects involved on such short space/time scales, a microscopic analysis of their performances is a rather complicated task, and it is performed usually by numerical methods.

At the present time two main approaches, that we shall call stochastic and deterministic, respectively, are used for a theoretical investigation of the intrinsic noise of semiconductor devices. Within a stochastic approach, fluctuations result from simulation of the expected behavior of carriers in a device by making use of random events generated with given probabilities. Accordingly, this approach is based on various procedures of solution of a kinetic task, grounded on the Boltzmann transport equation (BTE), where the motion and interactions of charge carriers inside a device are simulated by the Monte Carlo (MC) method [3–16]. From one hand, this approach allows one to account in the most comprehensive way for the peculiarities of carrier dynamics in external and self-consistent internal fields,

for the device design, etc. From another hand, it contains in a natural way the source of fluctuations caused by the stochastic nature of the quantum-mechanical description of various interactions. The main drawback of this approach is that it allows one to obtain mainly integral characteristics of the intrinsic noise of a device without offering a detailed spatio-temporal picture of the processes responsible for the noise. As a consequence, to investigate the detailed behavior of the noise characteristics one often needs to construct additionally simple analytical models able to explain such a behavior.

The deterministic approach is based on the introduction of random forces of Langevin type as fluctuation sources and a further deterministic description of the linear response of the considered characteristics caused by these perturbations [17–20]. Within such an approach, the resulting response is represented as a sum over the device volume of local perturbations originated by the noise sources, which are transferred up to the device terminals by the so called *transfer fields*. The advantage of such an approach is that, contrary to the stochastic approach, it allows one to obtain a sufficiently detailed picture of the spatial origin of the intrinsic noise based on a separate description of the noise sources and the corresponding transfer fields. Moreover, most of the results can be formulated at an analytical level thus providing a comprehensive theoretical description of the general properties of electronic noise in semiconductor materials and devices. It should be emphasized that various models, at different levels of description of carrier transport and noise, can be used in bulk semiconductors and semiconductor devices. These models are formulated using different sets of microscopic or macroscopic equations, and, hence, different types of Langevin forces are required to introduce the carrier noise within this variety of models. In the framework of a classical description, the most comprehensive theory of carrier transport and noise is based on the Boltzmann-Langevin (BL) scheme [21–24]. Here, the Langevin force initiates the fluctuations of the distribution function caused by the stochastic nature of scattering events, i.e. the BL scheme is entirely equivalent to the stochastic solutions of the BTE obtained by MC methods. Moreover, due to the Markovian nature of scattering events, the spatio-temporal characteristics of such a noise source are the most simple, namely, the

Langevin force is $\delta-$correlated both in time and space thus implying that its spectral density is white [23–26]. However, it is rather difficult to perform noise calculations for real devices at the BL level due to the very large number of variables involved. Therefore, simplified transport models are often used for this sake. All simplified transport models, such as BTE with relaxation-time approximation [27–29], various hydrodynamic [30–40] and drift-diffusion models [41–57] can be derived from the BTE under some assumptions. Correspondingly, the Langevin forces and their spectral densities appropriate to these simplified transport models, strictly speaking, should be obtained from the BL scheme. However, in practice, for simplified models the Langevin forces are usually postulated *ad hoc* rather than being rigorously derived from the BL scheme, what implies that the transfer fields and the corresponding Langevin forces are treated in such a case as independent quantities.

One of the first and still now widely used deterministic approach is the impedance field (IF) method proposed in a seminal paper by Shockley, Copeland and James in 1966 [58] for the description of the spectral density of voltage fluctuations, $S_U(f)$, of two-terminal devices. The IF method follows the two step procedure outlined above. First, it is supposed that the source of fluctuations is given by local fluctuations of the conduction current density treated as uncorrelated in space. Second, the transfer field for this noise source is constructed euristically by determining the perturbation of the voltage drop between the structure terminals initiated by a perturbation of the current inside the structure, virtually introduced by two additional local contacts necessary to initiate the local fluctuation. Finally, the formula which relates the spectral density of voltage fluctuations $S_U(\omega)$ with the local spectral density of current fluctuations inside the structure takes the form:

$$S_U(\omega) = \int_0^L |\nabla Z(x', \omega)|^2 S_{jj}(x', \omega) dx' \tag{1}$$

where L is the device length, $\omega = 2\pi f$ the circular frequency, $\nabla Z(x', \omega)$ the impedance field, which relates perturbations of voltage with local current perturbations at point x', $S_{jj}(x', \omega)$ the spectral density of the uncorrelated local fluctuations of conduction current, and for the sake of simplicity a one-dimensional geometry is considered.

A rigorous mathematical proof of the IF method as a special case of the more general Langevin approach was given in Refs. [59–62]. There it was shown that the IF is related to the Green functions which describe the response of the voltage drop at the terminals to local δ-like fluctuations of the conduction current. In other words, within this formalism the Langevin force is the conduction current fluctuations. The next standard approximation is the assumption that conduction current fluctuations are spatially uncorrelated (and, strictly speaking, fluctuations of carrier concentration are absent). In this case the spectral characteristic of the IF noise source at frequency f, i.e. $S_{jj}(x, f)$, was usually represented as $e^2 n_s(x) S_v(x, f)$, where e is the electron charge, $n_s(x)$ the local free carrier concentration, and $S_v(x, f)$ the spectral density of local velocity fluctuations. Since the latter quantity is related to the frequency dependent diffusion coefficient as $4D(x, f) = S_v(x, f)$ [58, 62–66], this noise source is usually called diffusion noise source. Let us refer to this approach as the conventional IF method.

Successively, the IF method was further developed to include generation-recombination noise [50], what implies, by definition, the fluctuations of carrier concentration, and 1/f noise [49, 66]. Then, a more detailed development of the conventional IF method in the framework of the spectral representation was performed by including the analysis of the transfer impedance properties [64–66], by extending the IF to transistors [45, 67, 68] and multi-port devices [47, 48], by considering periodical large-signal operation in frequency multipliers and mixers [69–71, 51], by including the IF into device simulators [72], etc. The reasonable simplicity of the IF method has led to many implementations for noise calculations in semiconductor devices, especially within the drift-diffusion approximation [42–51], where even analytical approaches have been developed to analyse various semiconductor structures [52–57].

Together with the spectral representation of the IF, the time-domain formulation, which demonstrates more evidently the advantages of the Green-function formalism, has been developed. To this purposes we mention the hydrodynamic approach based on the velocity and energy conservation equations [34, 35, 37], and the deterministic solution of the BTE

by the so called *scattered packet method* [73–76]. By using the IF calculations, for the case of n^+nn^+ Si structures it was demonstrated that all these main implementations of the IF method give the same results [77]. In so doing the time-domain representation provides the additional advantage of analyzing the spatio-temporal evolution of single fluctuations induced in different points of the device under test.

Let us emphasize once more that starting from the seminal paper of Shockley et al. [58], all the above mentioned extensions of the IF method were performed by using the same diffusion noise source, obtained under the assumption that conduction current fluctuations are spatially uncorrelated and that they can be represented finally through the spectral density of velocity fluctuations. However, from the point of view of the BL scheme, conduction current fluctuations can not be treated as a primitive noise source, since within this scheme they in turn already result from the spatio-temporal evolution of the initial primitive perturbations originated by instantaneous scattering events. The frequency dependence of $S_v(x, f)$ is just the result of such a time evolution, depending on the material, applied voltage, etc. [78–88]. Therefore, since there is no $\delta(t)$-correlation in time for the conduction current fluctuations in the general case, there is also no $\delta(x)$-correlation in space for them. Thus, *the conventional IF formula contains the internal contradiction of having a frequency-dependent noise source but being local in space.*

The importance of spatial correlations between velocity fluctuations, while negligible for long semiconductor devices, is expected to increase significantly [89–96, 36] with the decrease of the device dimensions to length scales comparable with the carrier mean free-path, that is into the submicron size typical of modern microelectronics. Simple estimates show that in typical semiconductors, like GaAs, a spatial correlation can exist up to distances of $0.5 \div 2.0\ \mu m$ (average velocity of $1 \div 2 \times 10^5\ m/s$ multiplied by time interval of $1 \div 5\ ps$ corresponding to the usual duration of the correlation function of velocity fluctuations [78–88]). Indeed, such a correlation was observed in MC calculations for both homogeneous and submicron structures [95, 96, 36]. Therefore, for applying the IF method to sub-micron devices it is necessary to remove the above contradiction from the IF formula.

The aim of this review is to address the above issue by revisiting the physical background of the IF method and providing what we consider a necessary implementation of the method able to account effectively for non-local effects typical of deep submicron devices. Generally speaking, there are at least two ways to overcome the problem of accounting for the spatial correlation of conduction current fluctuations within the IF method. The first way is to generalize Eq. (1) to the case when the standard noise source, i.e. the conduction current fluctuations, includes the spatial correlation and their spectral power is described by two-point spectral density $S_{jj}(x_1, x_2, f)$. Since as we shall see below, only the velocity fluctuations must be accounted for in such a noise source and any fluctuations of concentration must be omitted, we shall refer to this way as the velocity fluctuation scheme. The second one is to reformulate the noise source content, i.e. to come back to the primitive noise source keeping its Markovian nature, and, hence, its $\delta-$correlation in time and space, as it is required by the BL scheme. We shall refer to this way as the acceleration fluctuation scheme.

The paper is organized as follows. In the first part, i.e. Sec. 2, we consider the velocity fluctuation scheme. The microscopic background of the IF method is outlined in Sec. 2.1 starting from the BTE with a Langevin source of fluctuations. In Sec. 2.2 a general relation between the spectral density of voltage fluctuations and the two-point spectral density of conduction current fluctuations responsible for the noise is derived in the framework of the linear response theory by using the Green-function approach. The main features of the cross-correlation function of velocity fluctuations are investigated in Sec. 2.3 within an analytical approach. To illustrate the general trends, Sec. 2.4 presents numerical results calculated by MC and HD approaches for both over- and sub-micron structures. Major conclusions related to the use of the velocity fluctuation scheme and alternative perspectives for noise calculations in submicron devices are summarized in Sec. 2.5.

In the second part of the paper, i.e. Sec. 3, we consider the acceleration fluctuation scheme. The HD Langevin forces corresponding to this scheme are obtained from the BL approach in Sec. 3.1. Section 3.2 introduces the Green functions appropriate for the solu-

tion of the linear response problems. As an example of possible applications of the general procedure, the cross-correlation functions of conduction current fluctuations used in Sec. 2 as the noise source for the conventional IF method are calculated here directly using the HD-Langevin approach. A comparison with the results of Sec. 2 is then carried out. Various operation modes, corresponding to the various conditions in which the structure can be connected with the external circuit, are considered in Sec. 3.3. The generalized impedance and admittance field methods for the voltage- and current-noise operation modes, respectively, are constructed here in the framework of a unified formalism. To illustrate the general trends, Sec. 3.4 presents numerical results calculated by the HD-Langevin approach for the constant current, constant voltage and circuit operation modes of the same submicron structure of Sec. 2. The spatio-temporal evolution of single fluctuations, the resulting temporal and spectral behavior of the generalized impedance and admittance fields, etc. are then compared for different operation modes. The advantages of the procedure are also illustrated by noise calculations in more complicated devices such as GaAs n^+n- Schottky-contact and Si p^+n structures. The major conclusions related to the use of the acceleration fluctuation scheme for the calculation of noise in submicron devices are given in Sec. 3.5. Final conclusions and future perspectives are given in Sec. 4. An appendix presents some analytical considerations concerning the spatio-temporal evolution of single fluctuations initiated in a non-homogeneous structure by the primitive noise source when operating under various operation modes.

II. VELOCITY FLUCTUATION SCHEME

The main objective of this section is to present and illustrate the application of the velocity fluctuation scheme for noise calculations in submicron semiconductor devices when the length of the active region of the device is comparable with the typical lengths of spatial correlation of conduction current fluctuations. For this sake, we shall use mostly the time-domain representation of the correlation and response functions. The advantage of such a

representation is that it directly accounts for the sequence of causes and effects by relating events on the spatio-temporal scales in a natural way. The sequential physical picture so provided allows one to check and validate the limits of applicability of the main assumptions which are introduced, a task which is sometimes difficult to carry out using the frequency domain representation only. As a prototype of submicron devices, where the effects related to spatial correlations are expected to be important, we will take n^+nn^+ structures of GaAs since for this material the correlation length is in the range $0.1 \div 0.5\ \mu m$ at room temperature [95, 96, 36] .

A. Microscopic background of the impedance field method

Let us briefly review the formal scheme used to construct the IF in the presence of spatial correlations of conduction current fluctuations. It should be emphasized that, in the original article [58] , the IF gave the voltage response to an impulsive local perturbation of the total current artificially introduced by two additional contacts placed at an infinitesimal distance. However, this way of proceeding was more phenomenological than physical. Therefore, the initial theory was reformulated [62] in terms of an IF which gives the voltage response to a conduction current perturbation when the total current is kept constant both in time and space. This implies just the change of the IF sign, keeping the same formalism of linear response theory but providing a more physical approach. Within this approach, the proper microscopic noise source, i.e. scattering events, is replaced by its consequence, i.e. fluctuations of conduction current. We remark that this latter quantity contains two contributions. The first comes from the velocity fluctuations directly originated by scatterings. The second comes from the fluctuations of the concentration induced by the same velocity fluctuations through the continuity equation. As a consequence, the same fluctuations are accounted for twice by the conduction current fluctuations since these include both velocity and concentration fluctuations and, hence, cannot be considered as the proper noise source. With this remark in mind, the objective of the present section is to derive the main equations

determining the IF method starting from the proper noise source, i.e. using the BTE when microscopic fluctuations are given by Langevin forces. The advantage of this approach is that it strictly separates the fluctuation source given by the Langevin force from its further evolution governed by the BTE.

In the framework of the microscopic approach based on the BTE for the hot-carrier distribution function in momentum $\mathbf{p}$ and real space, $f(\mathbf{p}, x, t)$, normalized to the local carrier concentration as $n(x,t) = \int f(\mathbf{p}, x, t) d\mathbf{p}$, fluctuations caused by scattering events can be described by an additional stochastic Langevin force $\xi(\mathbf{p}, x, t)$ δ-correlated both in time and space [17, 23, 26, 24]:

$$\hat{L} f(\mathbf{p}, x, t) = \xi(\mathbf{p}, x, t) \tag{2}$$

where $\hat{L}$ is the kinetic operator, which for the one-dimensional structures considered here takes the form:

$$\hat{L} = \frac{\partial}{\partial t} + v(\mathbf{p}) \frac{\partial}{\partial x} + eE(x,t) \frac{\partial}{\partial p_x} - \hat{S} \tag{3}$$

$v(\mathbf{p})$ being the carrier group velocity along the field in state $\mathbf{p}$ and $\hat{S}$ the collisional operator in the classical form which neglects two-particle interactions given by:

$$\hat{S}[f] = -\gamma(\mathbf{p}) f(\mathbf{p}, x, t) + \int W(\mathbf{p}, \mathbf{p}') f(\mathbf{p}', x, t) d\mathbf{p}' \tag{4}$$

where $W(\mathbf{p}, \mathbf{p}')$ is the probability per unit time for a carrier to be scattered from state $\mathbf{p}'$ into state $\mathbf{p}$ at point x, $\gamma(\mathbf{p}) = \int W(\mathbf{p}'', \mathbf{p}) d\mathbf{p}''$ is the total scattering rate from state $\mathbf{p}$ into all possible final states $\mathbf{p}''$, and $E(x,t)$ the self-consistent electric field which acts on carriers inside the structure. The electric field is described by macroscopic field equations which include the conservation law of the total current density, J, given by the sum of the conduction and displacement currents

$$J = j_{cond}(x,t) + \epsilon\epsilon_0 \frac{\partial E(x,t)}{\partial t} \tag{5}$$

and the Poisson equation

$$\frac{\partial E(x,t)}{\partial x} = \frac{e}{\epsilon\epsilon_0} [n(x,t) - N_d(x)] \tag{6}$$

Here $N_d(x)$ is the donor concentration profile, $j_{cond}(x,t) = e\int v(\mathbf{p})f(\mathbf{p},x,t)d\mathbf{p}$ the instantaneous local conduction current density, ϵ_0 the vacuum permittivity, and ϵ the static dielectric constant of the material. In the absence of Langevin forces, a joint solution of Eqs. (2) - (6) provides the steady-state distribution function, $f_s(\mathbf{p},x)$, and the stationary profile of the self-consistent electric field, $E_s(x)$. The conservation law for the total current reduces in this case to the trivial relation $J = j_{cond}(x) = const$. It should be stressed that, due to the presence of the self-consistent electric field, the system of Eqs. (2) and (6) is non-linear with respect to both $f(\mathbf{p},x,t)$ and $E(x,t)$.

Let us suppose that the fluctuations caused by the Langevin force are small enough to be described within a linear approximation. In this case the total distribution function and the electric field can be decomposed into the sum of a stationary term and a time dependent term, responsible for fluctuations, as:

$$f(\mathbf{p},x,t) = f_s(\mathbf{p},x) + \delta f(\mathbf{p},x,t) \tag{7a}$$

$$E(x,t) = E_s(x) + \delta E(x,t) \tag{7b}$$

By substituting Eqs. (7) into Eqs. (2) - (6), after linearization one obtains the following system of equations which describes fluctuations:

$$\hat{L}_s\delta f(\mathbf{p},x,t) + e\delta E(x,t)\frac{\partial}{\partial p_x}f_s(\mathbf{p},x) = \xi(\mathbf{p},x,t) \tag{8}$$

$$\epsilon\epsilon_0\frac{\partial\delta E(x,t)}{\partial t} + e\int v(\mathbf{p})\delta f(\mathbf{p},x,t)d\mathbf{p} = 0 \tag{9}$$

$$\epsilon\epsilon_0\frac{\partial\delta E(x,t)}{\partial x} - e\int \delta f(\mathbf{p},x,t)d\mathbf{p} = 0 \tag{10}$$

Here $\hat{L}_s$ is the kinetic operator given by Eq. (3) which includes only the static electric field, $E_s(x)$, i.e. the electric field with a frozen profile, instead of the dynamic $E(x,t)$.

In accordance with Eq. (8), the fluctuations of the distribution function $\delta f(\mathbf{p},x,t)$ are originated by two causes. The first is the microscopic noise source described by the Langevin force $\xi(\mathbf{p},x,t)$. The second is the macroscopic fluctuation of the self-consistent electric field $\delta E(x,t)$ induced by the microscopic source and described by the field equations (9) and

(10). To specify the noise source entering the IF from a microscopic point of view, the total fluctuations of the distribution function is conveniently represented by the sum of the above two contributions as:

$$\delta f(\mathbf{p}, x, t) = \delta f_{\xi}(\mathbf{p}, x, t) + \delta f_{E}(\mathbf{p}, x, t) \tag{11}$$

The first contribution can be written as:

$$\delta f_{\xi}(\mathbf{p}, x, t) = \int d\mathbf{p}_0 \int dx_0 \int_{-\infty}^{t} P_s(\mathbf{p}, x, \mathbf{p}_0, x_0, t - t')\xi(\mathbf{p}_0, x_0, t')dt' \tag{12}$$

where $P_s(\mathbf{p}, x, \mathbf{p}_0, x_0, t - t_0)$ is the Green function of the kinetic operator (3) with the frozen profile of $E(x, t) = E_s(x)$ and given by:

$$\hat{L}_s P_s(\mathbf{p}, x, \mathbf{p}_0, x_0, t - t_0) = \delta(t - t_0)\delta(x - x_0)\delta(\mathbf{p} - \mathbf{p}_0) \tag{13}$$

Let us note, that for the collisional term linear with respect to the distribution function (see Eq. (4)) the Green function given by Eq. (13) can be interpreted as the probability of finding a carrier at time t in the neighborhood of point x in state $\mathbf{p}$ under the condition that the carrier was at time moment $t_0 \leq t$ in the neighborhood of point x_0 in state $\mathbf{p_0}$ (the so called transition or conditional probability). The second contribution is given by Eq. (8) without the Langevin force as:

$$\hat{L}_s \delta f_E(\mathbf{p}, x, t) + e\delta E(x, t)\frac{\partial}{\partial p_x} f_s(\mathbf{p}, x) = 0 \tag{14}$$

¿From the field equations (9) and (10), which describe the spatio-temporal evolution of $\delta E(x, t)$, the rate of temporal variations of $\delta E(x, t)$ at a point x is given by the amplitude of conduction current fluctuations, $\delta j_{cond}(x, t) = e \int v(\mathbf{p})\delta f(\mathbf{p}, x, t)d\mathbf{p}$, and the spatial variations of $\delta E(x, t)$ are determined by the amplitude of concentration fluctuations, $\delta n(x, t) = \int \delta f(\mathbf{p}, x, t)d\mathbf{p}$. Since in the framework of the BTE formalism $\delta n(x, t)$ and $\delta j_{cond}(x, t)$ correspond to fluctuations of the zero and first moments of the distribution function, the separate description of the spatial and temporal variations of the electric field can be avoided by multiplying Eq. (10) by the stationary carrier velocity, $v_s(x) = \int v(\mathbf{p})f_s(\mathbf{p}, x)d\mathbf{p}/n_s(x)$ and adding the result to Eq. (9). Here $n_s(x) = \int f_s(\mathbf{p}, x)d\mathbf{p}$

is the stationary profile of carrier concentration. Taking into account Eq. (11), one obtains the equation which describes the propagation of the electric field perturbation inside the structure:

$$\epsilon\epsilon_0 \left[\frac{\partial}{\partial t} + v_s(x)\frac{\partial}{\partial x}\right] \delta E(x,t) + e n_s(x) \delta v_E(x,t) = -e n_s(x) \delta v_\xi(x,t) = -\delta\tilde{j}(x,t) \qquad (15)$$

where

$$\delta v_\alpha(x,t) = \frac{1}{n_s(x)} \int [v(\mathbf{p}) - v_s(x)] \delta f_\alpha(\mathbf{p},x,t) d\mathbf{p} \qquad (16)$$

is the fluctuation of the carrier average velocity induced by the self-consistent electric field ($\alpha = E$) or directly caused by the Langevin source ($\alpha = \xi$). As follows from Eqs. (15)-(16), only the part of the conduction current fluctuations directly related to the local fluctuations of the average velocity, namely, $\delta\tilde{j}(x,t) = e n_s(x) \delta v_\xi(x,t)$, can be considered as the proper noise source. Let us note, that Eq. (15) is in essence a macroscopic equation for the description of the electric field fluctuation and, hence, it is independent of both the choice of the deterministic model used to describe the propagation media (BTE, HD, DD, etc.) and the microscopic nature of the source, i.e. $\delta v_\xi(x,t)$.

The representation given by Eqs. (12) and (16) for the so called diffusion noise source $\delta\tilde{j}(x,t) = e n_s(x) \delta v_\xi(x,t)$ in the IF method allows one to establish the requirements that the noise source should fulfill in the general case of non-homogeneous systems. These requirements are rather general and independent of the choice for the transport model used to describe the medium. They can be summarized in the following two main points as:

(i) The conduction current fluctuations must contain only the contribution related to fluctuations of average velocity, the contribution related to the induced fluctuations of concentration must be omitted.

(ii) The conduction current fluctuations and their spatial dependence must be obtained in the absence of fluctuations of the self-consistent electric field which acts on carriers. Accordingly, the spatial profile of the electric field must be frozen, thus corresponding to the non-fluctuating stationary value $E_s(x)$.

In the general case of the BTE, the IF is constructed using the two-component Green-function of the equation system (14)-(15):

$$\mathbf{G}(\mathbf{p}, x, x_0, t - t_0) = [G_E(x, x_0, t - t_0), G_{f_E}(\mathbf{p}, x, x_0, t - t_0)] \tag{17}$$

which at $t = t_0$ satisfies the condition: $\mathbf{G}(\mathbf{p}, x, x_0, 0) = [\delta(x - x_0), 0]$. Here the field component, G_E, describes the electric field fluctuations induced by a unit impulsive fluctuation $\delta\tilde{j} \propto \delta(t - t_0)\delta(x - x_0)$ occurred in point x_0 at time $t_0 < t$. The medium component, G_{f_E}, describes the fluctuations of the carrier distribution function induced by the electric field perturbations. Due to its macroscopic nature, the field equation (15) for the Green-function (17) has no direct connection with the deterministic model used to describe the medium. As a consequence , this section confirms that, in the framework of the IF method, the calculation of voltage fluctuations between the structure terminals can be reduced to two independent tasks, namely:

(i) the construction of the formula for the IF,

(ii) the determination of the conduction current fluctuations related to velocity fluctuations only under a frozen electric field profile.

In the following section we will construct the explicit formula for the spectral density of voltage fluctuations on the basis of the two above tasks. To this purpose we make use of the concept of the transfer impedance originally introduced in Ref. [62] and of the two-point noise source spectral strength obeying the requirements stated above. Being an implementation of the conventional transfer impedance method, we shall call it *extended transfer impedance formula*. Since the derivation of this formula is a macroscopic task, the considerations which will be further carried out in Sec. 2.2 are valid for all medium equations formulated in terms of the BTE, HD and DD approaches. A difference can exist merely for the description of the medium component G_{f_E} of the Green-function (17) and, hence, for perturbations of the average carrier velocity in Eq. (15) induced by the electric field, which are not used explicitly for the derivation of the extended transfer impedance formula.

B. Extended transfer impedance method

By using the field component G_E of the Green-function in Eq. (17), the fluctuation of the electric field originated by all previous fluctuations of the conduction current component $\delta\tilde{j}$ associated only with velocity fluctuations under the frozen electric field, can then be written as:

$$\delta E(x,t) = \int_{-\infty}^{t} dt' \int_{0}^{L} dx' G_E(x,x',t-t')\delta\tilde{j}(x',t') \tag{18}$$

By integrating Eq. (18) over the whole device and after the replacement $t - t' = \tau$, one obtains for the voltage fluctuation between the structure terminals:

$$\delta U(t) = \int_{0}^{L} dx' \int_{0}^{\infty} d\tau R(x',\tau)\delta\tilde{j}(x',t-\tau) \tag{19}$$

where

$$R(x',\tau) = -\int_{0}^{L} G_E(x,x',\tau)dx \tag{20}$$

is the voltage response function, i.e. the time-domain representation of the IF, which provides the additive contributions of the source $\delta\tilde{j}(x',t')$ at points x' into the total value of $\delta U(t)$. We remark that $G_E(x,x',t)$ corresponds to the time domain representation of the transfer IF, originally introduced in the frequency domain [62].

¿From the Wiener-Khintchine theorem the spectral density of voltage perturbations, $S_U(\omega)$, is given by the Fourier transformation of the autocorrelation function of voltage fluctuations as:

$$S_U(\omega) = 2\int_{-\infty}^{\infty} \overline{\delta U(t)\delta U(t+s)}^{t} exp(i\omega s)ds \tag{21}$$

where the bar with super-index t corresponds to time-averaging over t. Substitution of Eq. (19) into Eq. (21) gives:

$$S_U(\omega) = 2\int_{-\infty}^{\infty} exp(i\omega s)ds \int_{0}^{L}\int_{0}^{L} dx'dx'' \int_{0}^{\infty}\int_{0}^{\infty} d\tau' d\tau''$$

$$R(x',\tau')R(x'',\tau'')\overline{\delta\tilde{j}(x',t-\tau')\delta\tilde{j}(x'',t+s-\tau'')}^{t} \tag{22}$$

Since under stationary conditions the autocorrelation function depends only on the time $u = s + \tau' - \tau''$, substitution of $s = u - \tau' + \tau''$ and $t' = t - \tau'$ into Eq. (22) gives:

$$S_U(\omega) = \int_0^L \int_0^L dx' dx'' \left[\int_0^\infty d\tau' exp(-i\omega\tau') R(x', \tau')\right] \left[\int_0^\infty d\tau'' exp(i\omega\tau'') R(x'', \tau'')\right] \left[2\int_{-\infty}^\infty exp(i\omega u) C_{\delta\tilde{j}}(x', x'', u) du\right] \tag{23}$$

where

$$C_{\delta\tilde{j}}(x', x'', s) = \overline{\delta\tilde{j}(x', t)\delta\tilde{j}(x'', t-s)}^t \tag{24}$$

is the appropriate correlation function of the current density fluctuations entering the IF method.

In accordance with the general theory, the first and second integrals in square brackets in the r.h.s. of Eq. (23) correspond to the IF and its complex-conjugate at points x' and x'', respectively:

$$\nabla Z(x', \omega) = \int_0^\infty d\tau' exp(-i\omega\tau') R(x', \tau') \tag{25}$$

Therefore, the final IF formula takes the form:

$$S_U(\omega) = \int_0^L \int_0^L dx' dx'' \nabla Z(x', \omega) \nabla Z^*(x'', \omega) K(x', x'', \omega) \tag{26}$$

where $K(x', x'', \omega)$ is the two-point spectral density of the noise source given by

$$K(x', x'', \omega) = 2\int_{-\infty}^\infty C_{\delta\tilde{j}}(x', x'', s) exp(i\omega s) ds \tag{27}$$

We stress that Eq. (26) can be obtained (and in fact it was in Ref.[64]) from heuristic arguments in a straightforward manner. However, it is the physical meaning of the two-point spectral density which, to our opinion, stays as non-trivial point and which in this paper receives a rigorous definition. Taking into account the symmetry of $C_{\delta\tilde{j}}(x', x'', s)$ the following transposition property is satisfied:

$$C_{\delta\tilde{j}}(x', x'', s) = C_{\delta\tilde{j}}(x'', x', -s) \tag{28}$$

and the noise source can be rewritten as:

$$K(x', x'', \omega) = 2[S_j(x', x'', \omega) + S_j^*(x'', x', \omega)] \tag{29}$$

where

$$S_j(x', x'', \omega) = \int_0^\infty C_{\delta\tilde{j}}(x', x'', s) exp(i\omega s) ds \tag{30}$$

is the complex Fourier transform of the cross-correlation function $C_{\delta\tilde{j}}(x', x'', s)$.

It is evident that for spatially homogeneous systems, independently of the presence or not of current flux, the correlations exhibit translational symmetry. Moreover, in the absence of current flux the cross-correlation functions in the homogeneous case are identical for forward and reverse times, s and $-s$, respectively. In this case, from Eq. (27) one directly recovers the well known formula for the spectral density of current fluctuations given by the cosinus-transformation of the corresponding cross-correlation functions [58, 62, 64, 90]. However, in general the noise source $K(x', x'', \omega)$ must be described by a complex quantity, whose imaginary part reflects the violation of the spatio-temporal symmetry. Of course, the symmetry property of the cross-correlation function given by Eq. (28) guarantees real values of $S_U(\omega)$. This is easily proved by writing explicitly the two terms of Eq. (26) corresponding to pairs (x', x'') and (x'', x') and then summing them.

C. Spatio-temporal correlation of velocity fluctuations

In this section the correlation function of current fluctuations is analyzed at a kinetic level. First we investigate the general relations with the single carrier distribution function and related quantities. Then, we develop a simple relaxation model able to provide analytical results useful for a rigorous physical interpretation of several intriguing features related to the spectral density of current fluctuations.

1. Kinetic analysis

In the framework of the Boltzmann Langevin approach, the correlation function of the conduction current fluctuations associated with only velocity fluctuations can be obtained

from Eqs. (12) and (16) as:

$$C_{\delta\tilde{j}}(x',x'',s) = \overline{\delta\tilde{j}(x',t)\delta\tilde{j}(x'',t-s)}^{t}$$

$$= e^2 \int d\mathbf{p}' \int d\mathbf{p}''[v(\mathbf{p}') - v_s(x')][v(\mathbf{p}'') - v_s(x'')]C_{\delta f_\xi}(\mathbf{p}',x',\mathbf{p}'',x'',s) \tag{31}$$

where

$$C_{\delta f_\xi}(\mathbf{p}',x',\mathbf{p}'',x'',s) = \overline{\delta f_\xi(\mathbf{p}',x',t)\delta f_\xi(\mathbf{p}'',x'',t-s)}^{t}$$

$$= \int d\mathbf{p}'_0 \int d\mathbf{p}''_0 \int dx'_0 \int dx''_0 \int_0^\infty du' \int_0^\infty du'' P_s(\mathbf{p}',x',\mathbf{p}'_0,x'_0,u')$$

$$P_s(\mathbf{p}'',x'',\mathbf{p}''_0,x''_0,u'')\overline{\xi(\mathbf{p}'_0,x'_0,t-u')\xi(\mathbf{p}''_0,x''_0,t-s-u'')}^{t} \tag{32}$$

is the correlation function of the fluctuations of the carrier distribution function under stationary conditions represented in terms of the correlation function of the Langevin force given by [23, 26, 24]:

$$\overline{\xi(\mathbf{p},x,t)\xi(\mathbf{p}',x',t-s)}^{t} = \frac{1}{2}S_\xi(\mathbf{p},\mathbf{p}',x)\delta(x'-x)\delta(s) \tag{33}$$

where $S_\xi(\mathbf{p},\mathbf{p}',x)$ is the spectral density of the Langevin force.

By substituting Eq. (33) into Eq. (32) one obtains the correlation function of the fluctuations of the carrier distribution function:

$$C_{\delta f_\xi}(\mathbf{p},x,\mathbf{p}',x',s) = \int d\tilde{\mathbf{p}} \int d\tilde{x} P_s(\mathbf{p},x,\tilde{\mathbf{p}},\tilde{x},s)C_{\delta f_\xi}(\tilde{\mathbf{p}},\tilde{x},\mathbf{p}',x',0) \tag{34}$$

expressed through the co-variance of the fluctuations of the occupation numbers (i.e. the distribution function) in $(\mathbf{p},x)$-space

$$C_{\delta f_\xi}(\mathbf{p},x,\mathbf{p}',x',0) = \frac{1}{2}\int dx_0 \int d\mathbf{p}_0 \int d\mathbf{p}'_0 S_\xi(\mathbf{p}_0,\mathbf{p}'_0,x_0)$$

$$\int_0^\infty du P_s(\mathbf{p},x,\mathbf{p}_0,x_0,u)P_s(\mathbf{p}',x',\mathbf{p}'_0,x_0,u) \tag{35}$$

Under steady-state conditions, the co-variance function of the occupancy numbers fluctuations (35) is a statistical property of the carrier ensemble and does not depend on any

relaxation time of the system. In accordance with Ref. [24], for an ensemble of independent carriers described by the BTE with the collisional term given by Eq. (4), the above co-variance function can be represented in the form:

$$C_{\delta f_\xi}(\mathbf{p}, x, \mathbf{p}', x', 0) = f_s(\mathbf{p}', x)\delta(\mathbf{p} - \mathbf{p}')\delta(x - x') - f_s(\mathbf{p}, x)f_s(\mathbf{p}', x')/N \qquad (36)$$

where the first term in the r.h.s. (the diagonal part) describes the autocorrelation of a local fluctuation in the occupation numbers of a single state and the second term in the r.h.s. (the off-diagonal part) describes the presence in the system of cross-correlation fluctuations in the occupation numbers between different states. For an ensemble of independent particles the appearance of the latter term, i.e. of a two-particle correlation, comes from the conservation of the total number of particles in the system: $N = \int\int f_s(\mathbf{p}, x)d\mathbf{p}dx$ [17, 23]. In the case of an open system which undergoes a free exchange of particles with a thermal bath or which is a part of a large homogeneous system, the off-diagonal part in Eq. (36) vanishes [23, 24].

Let us stress that the co-variance function (36) is derived from general properties of statistical ensembles and contains the stationary distribution function only, any property of the Langevin force being absent. Nevertheless, Eqs. (35) and (36) are equivalent, since usually the spectral characteristics of the Langevin forces in the BTE are derived from Eq. (36) (or some other similar to it) thus making the BTE description of fluctuations equivalent to a stochastic approach. An example of this equivalence is provided by the method of the inverse kinetic operator, as it was elaborated in Ref. [23].

By substituting Eq. (36) into Eq. (34) one obtains the two-point correlation function of occupation numbers as:

$$C_{\delta f_\xi}(\mathbf{p}, x, \mathbf{p}', x', s) = P_s(\mathbf{p}, x, \mathbf{p}', x', s)f_s(\mathbf{p}', x') - f_s(\mathbf{p}, x)f_s(\mathbf{p}', x')/N \qquad (37)$$

As follows from Eq. (37) the correlation time is connected only with the evolution of a local fluctuation appeared at $s = 0$ in some state $(\mathbf{p}', x')$ and it is described by the single-carrier transition probability $P_s(\mathbf{p}, x, \mathbf{p}', x', s)$ (the first term in the r.h.s. of Eq. (37)). The two-particle cross-correlation of fluctuations (i.e. the second term in

the r.h.s. of Eq. (37)) does not evolve in time since the stationary distribution of carriers in the ensemble is invariant with respect to carrier motion during time s, i.e. $f_s(\mathbf{p},x) = \int dx_0 \int d\mathbf{p}_0 P_s(\mathbf{p},x,\mathbf{p}_0,x_0,s) f_s(\mathbf{p}_0,x_0)$ at any $s \geq 0$. By substituting Eq. (37) into Eq. (31) the term describing the two-particle cross-correlation vanishes, since it describes two-point cross-correlation of fluctuations of carrier concentration only rather than fluctuations in momentum space. As a consequence, one obtains the following expression for the cross-correlation function of fluctuations of the conduction current component associated with velocity fluctuations only:

$$C_{\delta\tilde{j}}(x',x'',s) = e^2 \int d\mathbf{p}' \int d\mathbf{p}''[v(\mathbf{p}') - v_s(x')][v(\mathbf{p}'') - v_s(x'')] P_s(\mathbf{p}',x',\mathbf{p}'',x'',s) f_s(\mathbf{p}'',x'') \tag{38}$$

The correlation function given by Eq. (38) can be interpreted as a function which describes spatio-temporal correlations of velocity fluctuations, $\Delta v = v(\mathbf{p}) - v_s(x)$, in the ensemble of independent particles spreading in x-space from a neighborhood of some point $x = x'$ where they were at the initial time moment $s = 0$. Since the correlation of velocity fluctuations between any pair of particles in the ensemble is absent, a contribution to $C_{\delta\tilde{j}}(x',x'',s)$ is given only by the autocorrelation of velocity fluctuations of the same single particle, $\Delta v(x',s)\Delta v(x'',0)$, when it moves in x-space from point x'' (where it was at $s = 0$) to some other point x' undergoing both the acceleration from the applied electric field and the variations of velocity from scattering events.

In accordance with Eq. (38), only at $s = 0$ the correlation function, $C_{\delta\tilde{j}}(x',x'',s)$ is δ-correlated in x-space:

$$C_{\delta\tilde{j}}(x',x'',0) = e^2 n(x'') < \Delta v^2(x'') > \delta(x' - x'') \tag{39}$$

In this case it is described by local variations of instantaneous velocity fluctuations, $< \Delta v^2(x'') >= \int d\mathbf{p}''[v(\mathbf{p}'') - v_s(x'')]^2 f_s(\mathbf{p}'',x'')/n_s(x'')$ of carriers in a neighborhood of point $x = x''$. Due to carrier spreading, at all successive time moments $s > 0$ $C_{\delta\tilde{j}}(x',x'',s)$ loses the locality and must be considered as distributed in some volume of x-space which increases with time.

For a further analysis of $C_{\delta\tilde{j}}(x', x'', s)$ we need to detail the description of the transition probability determining the carrier movement in $(\mathbf{p}, x)$-space. In the general case of the BTE with the collisional term given by Eq. (4), recalling Chamber path integral formulation [97] $P_s(\mathbf{p}, x, \mathbf{p}_0, x_0, s)$ can be represented as the sum of two contributions which describe the dynamic and the stochastic processes of the system, respectively:

$$P_s(\mathbf{p}, x, \mathbf{p}_0, x_0, s) = D(\mathbf{p}, x, \mathbf{p}_0, x_0, s)$$

$$+ \int d\mathbf{p}' \int d\mathbf{p}'' \int dx \int_0^s D(\mathbf{p}, x, \mathbf{p}'', x', s - t') W(\mathbf{p}', \mathbf{p}'') P_s(\mathbf{p}', x, \mathbf{p}_0, x_0, t') dt' \qquad (40)$$

The first term in the r.h.s. of Eq. (40) represents the dynamic (singular) contribution:

$$D(\mathbf{p}, x, \mathbf{p}_0, x_0, s) = \delta[x - \tilde{x}(\mathbf{p}_0, x_0, s)] \delta[\mathbf{p} - \tilde{\mathbf{p}}(\mathbf{p}_0, x_0, s)]$$

$$exp\left[- \int_0^s \gamma[\tilde{\mathbf{p}}(\mathbf{p}_0, x_0, t')] dt' \right] \qquad (41)$$

which describes the probability of free motion of a carrier without scatterings at $s > 0$ along the trajectory determined from the equations of motion:

$$\frac{d\tilde{x}}{ds} = v(\tilde{\mathbf{p}}), \quad \frac{d\tilde{\mathbf{p}}}{ds} = eE_s(\tilde{x}) \qquad (42)$$

under the condition that the carrier was at time $s = 0$ in point $\tilde{x}(\mathbf{p}_0, x_0, 0) = x_0$, $\tilde{\mathbf{p}}(\mathbf{p}_0, x_0, 0) = \mathbf{p}_0$. The second term in the r.h.s. of Eq. (40) represents the stochastic (regular) contribution of the transition probability and describes the effect of carrier scatterings.

It is evident that for scattering mechanisms occurring isotropically in momentum space the spatio-temporal correlation of carrier velocity fluctuations can be maintained during a single free flight only, since any correlations of velocity fluctuations are lost immediately just after the first scattering event. In this case a non-zero contribution into $C_{\delta\tilde{j}}(x', x'', s)$ comes only from the dynamical part of the transition probability (40), i.e. one obtains an expression similar to that derived in Ref. [90], when instead of the total transition probability, $P_s(\mathbf{p}', x', \mathbf{p}'', x'', s)$, only its dynamical part given by Eq. (41) for $D(\mathbf{p}', x', \mathbf{p}'', x'', s)$ is used. Let us note, that in contrast to Ref. [90], Eq. (38) determines the two-point correlation

function of the component of conduction current fluctuations associated with velocity fluctuations only rather than the total fluctuations of the conduction current which also include concentration fluctuations as it was implied in Ref. [90]. For scattering mechanisms occurring anisotropically in momentum space the correlation can be maintained during several successive free flights. In any case, the correlation will be lost after the time necessary for the complete relaxation of the initial momentum to take place. Thus, the main features of the spatio-temporal correlation of velocity fluctuations will be related to the dynamics of the carrier free motion in $(\mathbf{p}, x)$-space during the momentum relaxation time.

2. Relaxation time model

Let us consider a simple relaxation model allowing us to obtain an analytical expression for $C_{\delta\tilde{j}}(x', x'', s)$ which reflects the main features of the spatio-temporal correlation without detailing the scattering mechanisms. To this purpose we consider space homogeneous conditions within the relaxation time model. Accordingly, the electric field is independent of the x-coordinate and the collisional term in the BTE, $S[f]$, is of the simple relaxation form: $S[f] = -\gamma[f(\mathbf{p}, x, t) - n(x, t)f_0(\mathbf{p})]$. The latter is equivalent to a representation of the carrier transition rate from the state $\mathbf{p}'$ into the state $\mathbf{p}$ due to scatterings in the form: $W(\mathbf{p}, \mathbf{p}') = \gamma f_0(\mathbf{p})$ where γ is the net scattering rate independent of carrier initial momentum $\mathbf{p}'$, and $f_0(\mathbf{p})$ is the carrier distribution in momentum space under thermal equilibrium conditions, $\int f_0(\mathbf{p})d\mathbf{p} = 1$. In this case, having in mind a one-dimentional character of the problem, the dynamic part of the transition probability (41) takes the form:

$$D(p, p_0, x - x_0, s) = \delta[p - p_0 - eEs]\delta[x - x_0 - \frac{p_0}{m}s - \frac{eE}{2m}s^2]exp(-\gamma|s|) \qquad (43)$$

By substituting Eq. (43) into Eq. (38) and performing the integration over final states p in momentum space one obtains an expression for $C_{\delta\tilde{j}}(x' - x'', s)$, which contains explicitly the time dependence of velocity fluctuations determined by the actual value of the carrier

momentum $p = p'' + eEs$ along the free-flight trajectory:

$$C_{\delta\tilde{j}}(x' - x'', s) = e^2 e^{-\gamma|s|} \int dp''(\frac{p'' + eEs}{m} - v_s)(\frac{p''}{m} - v_s)\delta(x' - x'' - \frac{p''}{m}s - \frac{eE}{2m}s^2) f_s(p'') \quad (44)$$

Here $v_s = \int p f_s(p) dp/(mn_s) = eE/(\gamma m)$ is the average velocity, $f_s(p) = \gamma n_s \int_0^\infty f_0(p - eEs')exp(-\gamma s')ds'$ is the stationary distribution function, and the argument of the δ-function describes the actual position of a carrier on the free-flight trajectory in x-space under the condition that at time moment $s = 0$ the carrier was in point x'' with momentum p''.

As follows from Eq. (44), to find $C_{\delta\tilde{j}}(x' - x'', s)$ one must perform an integration of the velocity fluctuations over some initial local ensemble formed by carriers which are placed under stationary conditions in the neighborhood of point x'' and have momentum p'' distributed in accordance with $f_s(p'')$. At $s = 0$ the δ-function is independent of p'' and can be taken out from the integral in Eq. (44). In this case, the integration over p'' plays the role of a local averaging of velocity fluctuations in accordance with a stationary distribution of momenta. At $s > 0$ carriers start to spread in x-space so that the argument of the δ-function begins to depend on p''. Due to this reason, a non-zero contribution into the integral in Eq. (44) is given only by carriers which are able to be shifted from point x'' into point x' during time s. The momentum p'' of such carriers in point x'' must satisfy the condition: $p''(x' - x'', s) = m(x' - x'' - eEs^2/2m)/s$, for which the argument of the δ-function is equal to zero. Thus, the carrier motion in x-space transforms the local averaging over the carrier momentum at $s = 0$ into an instantaneous coordinate distribution of carriers which started their motion at $s > 0$. As a consequence, Eq. (44) takes the form:

$$C_{\delta\tilde{j}}(x' - x'', s) = e^2 e^{-\gamma|s|} \begin{cases} n_s < \Delta v^2 > \delta(x' - x'') & , s = 0 \\ \frac{1}{|s|^2}\left[(x' - x'' - v_s s)^2 - \left(\frac{eEs^2}{2m}\right)^2\right] g(x' - x'', s) & , s \neq 0 \end{cases} \quad (45)$$

where $g(x' - x'', s) = (m/|s|) f_s[p''(x' - x'', s)]$ is the instantaneous density distribution of coordinates of a carrier ensemble spreading in x-space. At any time moment $s > 0$ this distribution is normalized to the value of the local concentration at point x'' at $s = 0$: $\int g(x' - x'', s)dx' = n_s(x'')$. The damping of the correlation described by the exponential factor in Eq. (45) can be interpreted as a carrier leaving such a spreading ensemble due to

scatterings.

Figure 1 shows $C_{\delta\tilde{j}}(x'-x'',s)$ calculated from Eq. (45) as function of the dimensionless time $u = s\gamma$ and distance $d = (x'-x'')/eEm\gamma^2$ for the case when the ratio between the drift and thermal velocities, v_s and $v_T = \sqrt{kT/m_0}$, respectively, is given by $v_s/v_T = 0.7$. The shape of the spatial cross-correlation presented in Fig. 1 is due to the fact that the contribution to the two-point correlation between points x' and x'' comes only from carriers which, during time s, can cross a distance $x'-x''$ without scatterings. Firstly, in the appearance of the cross-correlation for $x' \neq x''$, this fact leads to a delay time which is the longer the larger is the distance between the two points (see curves 1,3,5 and 2,4,6). Secondly, this fact causes an asymmetry of $C_{\delta\tilde{j}}(x'-x'',s)$ with respect to the sign of $x'-x''$ due to the different properties of carrier motion in the forward ($x' > x''$, curves 1,3,5) and backward ($x' < x''$, curves 2,4,6) directions with respect to an applied electric field E.

From the above consideration it follows that the temporal and spatial correlations of velocity fluctuations can not be treated separately as two independent properties of fluctuations. Indeed, both time and space correlations reflect the same process of free motion of a carrier in the neighborhood of some point in x-space. In other words, the presence of a temporal dependence of the correlation function of the velocity fluctuations leads directly to a spatial correlation of velocity fluctuations. Nevertheless, such a separation is usually introduced to simplify the extended IF formula given by Eq. (26). For this sake, the noise source is approximated by the local spectral density of velocity fluctuations [58, 62, 64, 43–57, 34, 35, 37], which, in terms of the present notations, can be written as:

$$K(x',x'',\omega) \equiv 4\int_0^\infty cos(\omega s) C_{\delta\tilde{j}}(x'-x'',s)ds = e^2 n_s(x'') S_v(x'',\omega)\delta(x'-x'') \qquad (46)$$

where $S_v(x'',\omega)$ is the local spectral density of single-carrier velocity fluctuations in the neighborhood of point x''. In such a case, the problem of the two-point correlation is completely avoided, since by substituting Eq. (46) into Eq. (26) one recovers the well-known local formulation of the conventional IF method given in Eq. (1). Nevertheless, even in this homogeneous case it is evident the impossibility to transform the correlation function

$C_{\delta\tilde{j}}(x'-x'',s)$ into a local approximation at any $s>0$. Only at $s=0$ $C_{\delta\tilde{j}}(x'-x'',s)$ is a local quantity, as it follows from Eq. (39) for a general case or from Eq. (45) for a homogeneous system. At any further time moments $s>0$, the correlation function takes some volume in x-space, which increases with the increase of s. As a consequence, the validity of the local approximation of the IF formula given by Eq. (1) cannot be justified by the assumption of a local noise source (i.e. the approximation of Eq. (46)) as it is usually done in literature [58, 62, 64, 43–57, 34, 35, 37]. The validity of the approximation (1) is related to the dependence of the IF on the spatial coordinate and can be justified only in the case when the spatial variation of $\nabla Z(x,\omega)$ can be neglected on the scale of the noise source non-locality determined by the mean free path λ, that is, when the following condition is fulfilled in the neighborhood of a given point x : $\lambda\partial\nabla Z(x,\omega)/\partial x << \nabla Z(x,\omega)$. In this case, in Eq. (26) one can replace x' by $x''+\alpha$, and then perform the integration over α by assuming that $\nabla Z(x''+\alpha,\omega)\approx\nabla Z(x'',\omega)$ is practically constant along the integration length where $K(x'',x''+\alpha,\omega)\neq 0$. As a consequence, the spectral power of velocity fluctuations distributed in the neighborhood of the point x'' will be gathered into a single quantity and one obtains the local approximation (1) of the IF formula, where the single-particle spectral density of velocity fluctuations $S_v(x'',\omega)$ is given by the Fourier transform of the single-particle correlation function determined as a space-integral in the neighborhood of point x'' from the cross-correlation function $C_{\delta\tilde{j}}(x',x'',s)$:

$$C_v(x'',s)=\frac{1}{e^2 n_s(x'')}\int_\lambda C_{\delta\tilde{j}}(x''+\alpha,x'',s)d\alpha \tag{47}$$

For example, in the simple homogeneous model considered above, the latter quantity can be easily calculated by using $C_{\delta\tilde{j}}(x'-x'',s)$ given by Eq. (45). In this case Eq. (47) recovers the well-known result for the single-particle correlation function:

$$C_v(x'',s)=<\Delta v^2(x'')>exp(-\gamma s) \tag{48}$$

Let us stress once more that, despite of the fact that in Eq. (48) the amplitude of the correlation function is determined by the variance of instantaneous velocity fluctuations

$< \Delta v^2 >= \int (p/m - v_s)^2 f_s(p) dp$, strictly speaking the correlation function itself is not a local quantity. Thus, the representation of the spectral density of velocity fluctuations in the form given by Eq. (46) can be considered merely as a formal representation, since Eq. (46) is just a consequence of the validity of Eq. (1) rather than its reason.

Let us note, that the misleading interpretation of the correlation function of velocity fluctuations as a local quantity is usually connected with the fact that in the derivation of Eq. (48), or similar to it, the free motion of carriers is considered in momentum space only, and the carrier motion in x-space is not taken into account. The typical example is when in Eq. (44) the δ-function which describes the carrier position in x-space is omitted. As a consequence, the integration over the initial momentum gives directly Eq. (48). Such a derivation is usually justified by invoking spatial homogeneity of the considered system when, due to translational symmetry, all processes are independent of position in x-space. Indeed, in this case the relevant quantities describing the microscopic processes occurring in the system contain no dependence on the local coordinate. However, this does not mean that all these quantities can be considered as locally-defined from a microscopic point of view.

Moreover, by neglecting the carrier motion in x-space one also neglects implicitly the local fluctuations of carrier concentration $\delta n(x,t)$. As a consequence, when calculating the conduction current fluctuations, $\delta j_{cond} = e n_s \delta v + e v_s \delta n$, the part of the fluctuations related to concentration fluctuations is omitted, since $\delta n = 0$. This leads sometimes to a somewhat misleading use of the term "conduction current density fluctuations" for the description of the noise source (see for example [62, 23, 98–100]) since in this case one accounts only for the part of conduction current fluctuations associated with velocity fluctuations, $\delta \tilde{j}_{cond} = e n_s \delta v$. On the other hand, as it was shown in Sec. 2.1, only this part contributes to the noise source in the framework of the IF method. Therefore, we argue that despite of an erroneous use of the term "conduction current density fluctuations" the omission of carrier motion in x-space compensates such an error and the final results for the noise source become correct by a fortuitous coincidence.

D. Numerical results

In the general case of inhomogeneous structures, analytical expressions for the hot-carrier steady-state distribution and Green-functions are not available and the hot-carrier transport and noise must be analyzed by numerical procedures. In doing so, to use the extended IF formula for voltage noise calculations given by Eq. (26), one should follow a three-step procedure:

(i) solve the stationary hot-carrier transport for the device under test taking into account the self-consistent electric field and find the spatial profile of relevant quantities such as $n_s(x)$, $E_s(x)$, etc. for a given value of the total current J_0 flowing through the structure;

(ii) solve the small-signal response problem under the same conditions, obtain the Green function $G_E(x', x'', \tau)$ and, hence, the IF by using Eqs. (20) and (25);

(iii) obtain the noise source and find the cross-correlation function of velocity fluctuations, $C_{\delta\tilde{j}}(x', x'', s)$, in the presence of the static electric field profile obtained at step (i).

The tasks (i) and (ii) can be solved by any deterministic procedure based on DD, HD, and BTE approaches. For this sake, the HD approach, which combines the simplicity of the DD and accounts for dynamic effects occurring at a kinetic level such as velocity overshoot, non-local heating of carriers, etc. inherent to the submicron length scale, has been proved to be quite satisfactory. It should be underlined that, to obtain the linear response in the framework of numerical methods, it is not necessary to linearize the main equations. For this sake it is sufficient to introduce a small perturbation into the whole system of equations and then to extract the system response from a full time-dependent solution. Therefore, since the linear task (ii) can be considered as a particular case of the more general non-linear task (i), it is preferable to formulate the corresponding HD approach in the time-domain representation from the very beginning. This allows us to use the same HD approach for the solution of both the stationary hot-carrier task (i) and the non-stationary linear task (ii).

In the framework of the transfer impedance method it is often preferable to take the local electric field as the relevant electrical variable [62]. In this way, to construct the

corresponding macroscopic part of the mathematical model, the Poisson equation (6) is rewritten as:

$$n(x,t) = N_d(x) + \frac{\epsilon\epsilon_0}{e}\frac{\partial E(x,t)}{\partial x} \tag{49}$$

After the substitution of Eq. (49) into Eq. (5) with the conduction current defined as $j_{cond}(x,t) = en(x,t)v(x,t)$ we obtain an equation for the electric field $E(x,t)$ in the form:

$$\frac{\partial E}{\partial t} + v\frac{\partial E}{\partial x} + \frac{e}{\epsilon\epsilon_0}vN_d - \frac{1}{\epsilon\epsilon_0}J_0 = -\frac{1}{\epsilon\epsilon_0}\delta\tilde{j}(x,t) \tag{50}$$

where, in accordance with Eq. (15), the r.h.s. already contains the noise source given by velocity fluctuations.

To close the macroscopic system of equations it is necessary to determine the average velocity of carriers. To this end we describe here the carrier transport in the framework of the conservation equations for carrier velocity $v(x,t)$ and mean energy $\varepsilon(x,t)$ written in the form [101, 102, 35, 37]:

$$\frac{\partial v}{\partial t} - eEm^{-1} + v\nu_v + v\frac{\partial v}{\partial x} + \frac{1}{n}\frac{\partial}{\partial x}(nQ_v) = 0 \tag{51}$$

$$\frac{\partial \varepsilon}{\partial t} - eEv + (\varepsilon - \varepsilon_{th})\nu_\varepsilon + v\frac{\partial \varepsilon}{\partial x} + \frac{1}{n}\frac{\partial}{\partial x}(nQ_\varepsilon) = 0 \tag{52}$$

Equations (51) and (52) contain five energy parametric dependencies, namely, the average of the reciprocal effective mass m^{-1} in the direction of the electric field E, the velocity and energy relaxation rates, ν_v and ν_ε, the variance of velocity-velocity fluctuations, $Q_v =< \delta v^2 >_0$ and the covariance of velocity-energy fluctuations, $Q_\varepsilon =< \delta v \delta\varepsilon >_0$, where brackets mean average over the hot-carrier distribution function in momentum space and the subscript 0 indicates steady-state conditions corresponding to constant and homogenous applied electric field. All the parameters are assumed to depend only on the local mean energy, and as such they can be obtained from a stationary MC simulation of the bulk semiconductor [101, 102, 35].

Equations (49) - (52) constitute a closed set which allows one to both: calculate the steady-state characteristics for a given value of J_0, and investigate the spatio-temporal evolution of the various perturbations responsible for the electronic noise in the structure. First

of all, by neglecting the noise source in the r.h.s. of Eq. (50) the stationary profiles of the electric field, concentration, velocity and energy are calculated for a given constant value of the total current J_0 flowing through the structure. Since under constant-current operation a stationary stable state of the system always exists, after some transient all the relaxation processes will terminate and the stationary profiles $E_s(x)$, $n_s(x)$, $v_s(x)$ and $\varepsilon_s(x)$ are determined.

In full analogy with Eq. (17), in the framework of Eqs. (49) - (52) the linear response of the system is described by the 4-component Green function:

$$\mathbf{G}(x, x', t - t') = [G_E(x, x', t - t'), G_n(x, x', t - t'), G_v(x, x', t - t'), G_\varepsilon(x, x', t - t')] \quad (53)$$

where the medium response is described by the average-velocity and mean-energy components, G_v and G_ε, respectively. For convenience, we introduce here also the Green function component G_n, which describes the concentration evolution. (Strictly speaking G_n is not necessary, since in accordance with Eq. (49) it can be directly obtained by taking the spatial derivative of the field component G_E.) By definition, the Green-function (53) describes the system response to a perturbation of the steady state given by:

$$\delta\tilde{j} = \Delta\delta(x - x_0)\delta(t) \quad (54)$$

Δ being an appropriate amplitude. By assuming that the initial perturbations are small enough to justify linearization, the difference between the time-dependent perturbed solution and the steady-state solution of the unperturbed system gives the Green-function. For example, for the field component of the Green function one obtains:

$$G_E(x, x_0, t) = \frac{\delta E(x, x_0, t)}{\Delta} = \frac{E(x, x_0, t) - E_s(x)}{\Delta}, \quad t \geq 0 \quad (55)$$

which describes the spatio-temporal evolution of the electric field perturbation caused by the local perturbation of the conduction current at point x_0. Similar equations hold for another components of the full Green function (53). In accordance with Eq. (54) the initial condition for the full Green function can be written as: $\mathbf{G}(x, x', 0) = [-\delta(x - x')/(\epsilon\epsilon_0), -(1/e)\partial\delta(x -$

$x')/\partial x, 0, 0]$. For the numerical solution of Eqs. (49)-(52), the initial perturbations of the steady-state profiles are given by the product $\mathbf{G}(x, x', 0)\Delta$. In practical calculations, the spatial-dependence of the initial perturbation can be taken, for example, of Gaussian form:

$$\delta(x - x_0) = \frac{1}{\gamma_0 \pi^{\frac{1}{2}}} exp\left[-\frac{(x - x_0)^2}{\gamma_0^2}\right] \tag{56}$$

where γ_0 is the semi-width of the perturbation which corresponds usually to $2 \div 10$ meshes with a typical value of about 1000 meshes for the whole length of the structure.

It is worthwhile to emphasize that the above procedure of calculations for the Green function (53) represents a full analog of Eqs. (14) and (15) formulated in terms of the distribution function fluctuations. Indeed, a joint linearization of Eqs. (49) and (50) gives directly Eq. (15). In turn, the coupled fluctuations of the average velocity and mean energy are now governed by the linearized Eqs. (51) and (52), which at the HD level correspond to the BTE-representation of Eq. (14).

In the framework of the HD approach described above by Eqs. (49)-(52), the noise source does not enter Eqs. (51) and (52), which describe the medium and give only the response induced by a source of fluctuations, and must be determined independently from other means. Therefore, to solve the noise problem (i.e. task (iii)) we make use the stochastic MC approach, which, by including microscopic fluctuations in a natural way, allows one to calculate the spatio-temporal cross-correlation function of conduction current fluctuations related to velocity fluctuations, $C_{\delta\tilde{j}}(x', x'', s)$, as described in detail in Refs. [95, 96].

To better illustrate the common points and differences between the extended and conventional IF method, in the following we apply the theoretical procedure previously described in parallel to the cases of sub- and over-micron structures. As an example of a submicron device we choose a $0.21 - 0.3 - 0.39$ μm n^+nn^+ GaAs structure with doping levels $n = 5 \times 10^{15}$ cm^{-3} and $n^+ = 10^{17}$ cm^{-3} at $T = 300$ K and with an applied voltage of 0.5 V. For the HD modeling the structure is subdivided into 900 meshes with a uniform spatial step of 1 nm. In the case of MC simulations the structure is subdivided into 90 meshes of 10 nm each. As an example of a long device, we simulate a $0.5 - 7.5 - 0.5$ μm n^+nn^+ GaAs structure with

doping levels $n = 2 \times 10^{14}$ cm^{-3} and $n^+ = 10^{16}$ cm^{-3} at $T = 300$ K and with an applied voltage of 8.0 V. The value of the voltage applied to the long structure is taken in such a way that the maximum mean energy of electrons and the maximum electric field achieved under these conditions are similar to those of the short structure.

For the long structure the usual approximation of the local noise source given by Eq. (46) with $S_v(x,\omega)$ obtained from the HD approach as described in Ref. [37] is used.

For the short structure, in order to calculate the spatially cross-correlated noise source we proceed as follow.

(i) In the MC simulation the stationary electric field profile is evaluated by averaging over an initial time period in which the field is updated at each time step by solving the time dependent Poisson equation.

(ii) The Poisson solver is switched off and the electric field profile is kept constant in time. Under these conditions the structure is subdivided into 30 cells of length $L_n = 300$ Å to calculate the noise sources $\delta\tilde{j}(x,t)$. Accordingly, at each time step the number of carriers in each cell $N_n(t)$ and their velocities $v_{ni}(t)$ are recorded. At the end of the simulation, the average number of carriers in each cell $\overline{N}_n$ and the average current flowing through the structure J are calculated. The fluctuation of the conduction current at time t in cell n is evaluated as

$$\delta j_{cond}(n,t) = \frac{e}{AL_n} \sum_{i=1}^{N_n(t)} v_{ni} - J \tag{57}$$

where A is the cross sectional area of the structure. This expression includes both number and velocity contributions to the fluctuations.

(iii) To eliminate the number contribution and obtain the current fluctuation to be included in the noise source of the IF, the following expression is used

$$\delta\tilde{j}(n,t) = \frac{e}{AL_n} \frac{\overline{N}_n}{N_n(t)} \sum_{i=1}^{N_n(t)} v_{ni} - J \tag{58}$$

The calculation of the cross-correlation of this fluctuation source between different cells $C_{\delta\tilde{j}}(n,m,s)$ is finally performed according to Eq. (24).

1. Stationary characteristics

Let us firstly show that MC and HD approaches give quite similar results for the stationary hot-carrier transport. As an example, Figs. 2(a) - 2(d) report the spatial profiles of the carrier concentration, drift velocity, mean energy and electric field calculated with the HD and MC approaches (solid lines and dots, respectively) for the submicron structure. The overall agreement between the HD and MC results, with quantitative discrepancies within $15 \div 20$ % at worst, is considered to be satisfactory. Both calculations evidence a pronounced velocity overshoot and a non-local heating of carriers in the n region. This is illustrated in Figs. 3(a) and (b), where the velocity and energy dependencies on the local electric field calculated with the HD approach for both the over- (curve 2) and sub-micron structures (curves 3) are compared with the same dependencies calculated with the MC method for bulk material (curve 1) and submicron structure (curve 4). Let us stress that the effects of velocity overshoot and non-local heating, while evident in the short diode, are negligible in the long structure, where both the velocity and energy dependencies on the internal electric field in the n region (upper branches of the curves) practically coincide with the analogous dependence in the bulk material.

2. Impedance field

In order to evaluate the IF, the submicron n^+nn^+ structure is divided into 30 cells of 300 Å each. An initial perturbation of the conduction current is introduced in the center of each cell, and the IF is calculated as described before. Figure 4 shows the spatial profiles of the real (curves 1,2) and imaginary (curves 3,4) parts of $\nabla Z(x_0, f)$ at frequencies of 25 (curves 1,3) and 225 GHz (curves 2,4), respectively. The results evidence a strong spatial dependence of the IF on the deep submicron length scale of about 0.1 μm. Furthermore, since all the microscopic processes taking place in the n-region (transit-time effects, etc.) are very fast (picosecond time scale), at low frequencies the IF is found to be practically independent

of frequency (for example, in the case of Fig. 4 the value of $Re\nabla Z(x_0, f)$ remains the same when $f = 0$ and 25 GHz). For comparison, Fig. 5 presents $|\nabla Z(x_0, f)|^2$ calculated for the long structure in the low-frequency range $0 \div 25$ GHz. By comparing Fig. 5 with Fig. 4, we conclude that a strong spatial dependence of the IF along the structure is also present in long devices. As usually (see for example Ref. [37]), the IF reaches a maximum in the near-cathode part of the n-region since, due to pronounced transit-time effects, this region provides the most significant contribution to the total voltage noise between the structure terminals. However, for the long structure significant variations of $|\nabla Z(x_0, f)|^2$ are detected only on a micron length scale, and one can neglect these variations on the deep submicron scale corresponding to the spatial correlation length of velocity fluctuations. Moreover, due to a relatively long transit-time of about 30 ps, the near-cathode peak of $|\nabla Z(x_0, f)|^2$ vanishes completely at frequencies above the transit-time frequency.

3. Spatially cross-correlated noise source

In accordance with Sec. 2.1, the evaluation of the spatially-correlated noise source must be performed with the static electric field profile $E_s(x)$ to avoid the electrical interaction between fluctuations through the self-consistent electric field, already taken into account by the IF. Furthermore, only the contribution due to velocity fluctuations must be taken into account. Thus, $C_{\delta\tilde{j}}(x', x'', s)$ has been calculated directly by using the MC technique [95, 96] through $\delta\tilde{j}(x, t)$ as originating only from velocity fluctuations (see also Eq. (58)). For comparison, some results will be presented also for the case of $\delta j_{cond}(x, t)$ when both electron number and velocity fluctuations are present (see Eq. (57)); the associated cross-correlation function will be denoted as $C_{\delta j}(x', x'', s)$.

In order to evaluate the spatial correlation of noise sources for the submicron structure of Fig. 2, the cross-correlation functions between different cells n, m, $C_{\delta\tilde{j}}(n, m, s)$ and $C_{\delta j}(n, m, s)$, are calculated with the MC technique and then introduced into Eq. (27) to obtain the noise source. To this end, we consider the structure divided into the same 30

cells of 300 Å each used to calculate the IF.

Examples of correlation functions are given in Figs. 6 to 8. The auto- ($C_{\delta\tilde{j}}(n,n,s)$) and cross-correlation ($C_{\delta\tilde{j}}(n,n+2,s)$) functions for $n = 10,\ 12,\ 14,\ 16$ are reported for positive times in Fig. 6(a) and (b), respectively. The general shape of the correlation functions is somewhat similar for different points in the n region. In particular we notice the absence of translational symmetry due to the strongly inhomogeneous conditions. At the initial time $s = 0$ the value of the autocorrelation function (see Fig. 6(a)) increases significantly when increasing the number of the cell, evidencing a strong heating of carriers when approaching the anode contact. Then, at increasing times it exhibits a monotonous decrease and vanishes for times longer than the ballistic transit time spent by the carriers in crossing the n-th cell. If a carrier undergoes a scattering event inside the n-th cell, the autocorrelation function tends to vanish. When compared with the analytical model in Eq. (45) of Sec. 2.3.2, we notice that the singularity of the autocorrelation function at $s = 0$ is absent due to integration over the whole n-th cell. The cross-correlation function (see Fig. 6(b)) is initially null because of the delay time necessary for the carriers to cross ballistically the $(n+1)$-th cell and appear in the $(n+2)$-th cell. Then, it peaks at the average time spent by carriers to cover the distance between cells without undergoing any scattering mechanism. At the longest times, it vanishes by showing in some cases a more or less pronounced negative tail at intermediate-long time scales. The observed spatial variations in Fig. 6(b) are connected mainly with the increase of carrier energy which takes place when approaching the anode contact and produces the two following consequences. First the maximum amplitude of the correlation function is found to increase considerably. Second the correlation functions exhibit a pronounced negative tail which is inherent to the presence of two time scales of relaxation (velocity and energy) typical of hot-electron conditions [78–88].

Figure 7 illustrates the spatial asymmetry of the cross-correlation functions $C_{\delta\tilde{j}}(n,m,s)$. The functions are reported for the positive time scale and refer to the cross-correlations between the cell $n = 13$ (near the center of the active region) and two following ($m = 15$,

17) and preceding ($m = 9$, 11) cells, also placed in the n-region. With respect to the central cell, the cross correlation functions display a rather asymmetric behavior in space mainly due to the presence of an electron flux through the structure. The general features of the cross-correlation are in good agreement with the results of the simple analytical model presented in Fig. 1. Due to the transposition property of Eq. (28), a spatial asymmetry implies a time asymmetry for a fixed pair of cells (n, m) with $n \neq m$. This is illustrated in Fig. 8 (a) which, in the whole time scale, shows the cross-correlation functions $C_{\delta\tilde{j}}(n, m, s)$ for $n = 10$ (in the active region near the maximum of the IF) and $m = 10$ (autocorrelation), 12, 14, 16, 18. Time reversal symmetry is preserved for the auto-correlation function, while it is broken for the cross-correlation functions. As expected, the correlation is more significant in the direction of carrier motion (which corresponds to $s > 0$ for $m > n$ and to $s < 0$ for $m < n$) and decreases as the distance between cells increases.

Relevant differences can be observed whether number and velocity fluctuations (see Fig. 8 (b)) or only velocity fluctuations (see Fig. 8 (a)) are taken into account in $\delta j(x, t)$. The time dependence of the correlation functions is quite similar for both cases. However, for $m > n$, $s > 0$ (and for $m < n$, $s < 0$), $C_{\delta j}(n, m, s)$ takes values much higher than those of $C_{\delta\tilde{j}}(n, m, s)$, which even exhibits a negative part. As a consequence, when compared with $C_{\delta\tilde{j}}(x', x'', s)$, $C_{\delta j}(x', x'', s)$ leads to overestimate the low-frequency value of the noise source.

Figure 9 shows an example of the two-point noise source $K(n, m, \omega)$ as a function of frequency for the pairs of cells $(n - 2, n)$ and $(n, n + 2)$, with $n = 14$. The noise source corresponds to conduction-current fluctuations associated only with velocity fluctuations. The complex character of the noise source due to the lack of time-reversal symmetry in the cross-correlation function is here clearly evidenced by the presence of a significant imaginary contribution. Furthermore, the differences exhibited by the curves 1, 3 with respect to 2, 4 evidence the absence of translational symmetry due to the spatial inhomogeneity of the structure.

We stress that the contribution of the imaginary part of the noise source to $S_U(\omega)$ takes place only under the simultaneous presence of: (i) an applied voltage, (ii) non-homogenous

conditions and, (iii) a frequency different from zero.

4. Spectral density of voltage fluctuations

The spectral density of voltage fluctuations between the terminals of the long structure calculated with different approaches and models is reported in Fig. 10. In the whole frequency range, an excellent agreement is found between the results obtained with MC and HD approaches (curves 1 and 2, respectively). Therefore, the standard IF procedure based on Eq. (1), which neglects two-point correlations, is confirmed to be quite sufficient to perform reliable noise calculations in long devices. Curve 3 reports $S_U(f)$ calculated with the HD approach but neglecting the frequency dependence of the local diffusion noise source, i.e. taking $S_v(x,\omega) = S_v(x,0)$. By comparing curves 1 and 2 with curve 3 in Fig. 10 we find that, even if the frequency dependence of the noise source is omitted, the $S_U(f)$ so obtained represents a very good approximation in the wide frequency range $f < 500\ GHz$, which is the range of main practical interest for over-micron structures. However, at the highest frequencies significant differences appear, thus indicating that a rigorous approach needs a frequency dependent spectrum for the spectral strength of the diffusion noise source.

The spectral density of voltage fluctuations between the terminals of the submicron structure is reported in Fig. 11. Here different curves refer to the following different ways of performing the calculations. Curve 1 is obtained with the MC technique, and thus represents the most rigorous result; curve 2 by using the IF method and neglecting spatial correlations according to Eq. (1) [i.e. taking a local noise source according to Eq. (46)]; curve 3 by using the IF method and including spatial correlations according to Eq. (26) with the noise source calculated from $C_{\delta\tilde{j}}(x',x'',s)$ (i.e. neglecting carrier number fluctuations); curve 4 is the same of curve 3 but with the noise source calculated from $C_{\delta j}(x',x'',s)$, (i.e. accounting for carrier number fluctuations). By naturally including all the processes responsible for the fluctuations and their correlations, the MC technique is taken to validate the calculations performed with the IF method and using different noise sources. The high frequency region

of the spectrum exhibits the plasma peak associated with the n^+ doping which is well reproduced by all calculations. By focusing on the low frequency region of the spectrum we draw the following conclusions. The best agreement with the MC results, excellent from a quantitative point of view, is obtained when spatial correlations are included in the IF calculation and only velocity fluctuations are considered in the noise source, as it follows from the general theory of the IF presented in Sec. 2.1. This agreement validates the formalism of the IF method presented here which includes the spatial correlations and confirms that the proper two-point noise source is that which contains only velocity fluctuations. When also concentration fluctuations are included in the noise source, the expected overestimation of the noise is found to be of relevant magnitude, over a factor of 6 for the present case. Finally, when spatial correlations are neglected and a local noise source is considered, the results of the corresponding IF method are found to overestimate the noise again but to a lower extent with respect to the previous case. Overall, a significant increase for about 100% with respect to MC calculations is found. Of course, the smaller the size of the active region of the device the larger the overestimation will be.

E. Concluding remarks

In this section we have presented a generalization of the conventional IF method able to account for non-local noise sources. The theoretical framework so developed has been validated by comparison with a MC technique. In the most general case, it is proved that the spatially correlated noise source is originated only by velocity fluctuations in the presence of frozen electric field conditions, and thus it does not include any fluctuations of carrier concentration.

For devices in the over-micron scale, when the characteristic length of the spatial cross-correlation becomes considerably smaller than that of the IF variation, the present generalization recovers the conventional formalism based on a local noise source. Under these conditions Eq. (26) goes to Eq. (1) with the integral form of the noise source

$e^2 n_s(x) S_v(x,\omega) = \int dx' K(x,x',\omega)$, which, by definition, contains velocity fluctuations only. For these devices the conventional IF method, with the spectral density of velocity fluctuations of bulk material taken as the local spectral strength of the diffusion noise source, is sufficient to calculate the voltage noise with a good accuracy. In this case the system of equations involved can be completely closed, since all the necessary physical quantities in Eq. (1) can be evaluated just in the framework of the HD model [33, 37]. Of course, as input parameters the static characteristics of bulk material (five quantities in the present case) must be obtained independently, e.g. from the kinetic description based on the BTE. Together with all the parameters of the model, the only basic assumption introduced here is that the spatial dependence of $S_v(x)$ is determined through the spatial dependence of the carrier mean-energy. From a physical point of view, this assumption seems to be quite natural, since the source of diffusion noise is scattering, whose intensity is mainly determined by the carrier energy.

For devices in the deep submicron scale, when the typical lengths of the device become comparable with or smaller than the correlation length of single-particle velocity fluctuations, the extended transfer IF method with the two-point noise spectrum must be used with the requirements we detailed above. Accordingly, the numerical solution of the noise problem can be carried out on a mixed HD-MC approach which provides the voltage noise with good accuracy. However, in so doing the main drawback is that the HD approach is no longer closed in the sense that the two-point spectral strength of the noise source is in general unknown. As such, it must be evaluated from an independent MC simulation (or some equivalent kinetic calculations) of the whole device in the presence of a frozen electric field. From a practical point of view this drawback is crucial. Indeed, in this case one needs to solve the transport problem at least twice: the first time by a deterministic approach to calculate the IF, and the second time by a stochastic approach to calculate the noise source. (It was our way to proceed to validate the mixed HD MC approach.) From a theoretical point of view such a situation becomes *unphysical*. As a matter of fact, the requirements of both the frozen electric field condition and the determination of only the velocity fluctuations reflect the

mathematical background of the formalism, rather than the physical nature of fluctuations. Of course, these requirements were implicitly used in the conventional IF approach when introducing the local noise source. However, by remaining physically plausible for large devices, this construction becomes rather artificial on submicron length scales, being in evident contradiction with the physical picture of microscopic fluctuations. In the attempt to calculate the voltage fluctuations of a real device, where the electric field self-consistently reflects any microscopic fluctuation, one finally needs to simulate the velocity noise under frozen electric field, a situation which cannot be physically realized at all in real devices. Thus, under these conditions it is much more convenient, if not mandatory, to solve the noise problem just once at a kinetic level, typically with the MC method, by calculating the voltage fluctuations in the device under constant-current operation [3, 7, 10, 12, 13, 15].

In essence, the above drawback originates from the dominant effects associated with the non-locality (both in time and space) of the noise source when going to the deep submicron length scale. As seen in this paper, non-locality is inherent to the velocity fluctuation scheme. Therefore, to keep the main advantages of the IF method, related to a separate description of the local noise source from its further transfer to the terminals, such a drawback can be in principle overcome by looking for a noise-source scheme alternative to that of velocity fluctuations. To this purpose, we have recently proposed the acceleration fluctuation scheme [38] for which the noise sources are by definition local in time and space thus exhibiting a white spectrum. The formal theory and the application of such alternative scheme will be the subject of the second part of the paper and is developed in Sec. 3.

III. ACCELERATION FLUCTUATION SCHEME

In the previous section we have considered the general approach of the conventional IF method pioneered by Shockley et al. [58] and its possible generalization within a *velocity fluctuation scheme*. It was found that the use of the conventional IF method becomes rather complicated and even to somewhat extent unphysical, when one wants to take into

account the spatial correlation of velocity fluctuations on length scales comparable to, or smaller than, the carrier mean-free-path. As a consequence, we are proposing an *acceleration fluctuation scheme* [38] which has the advantage of considering the proper noise sources δ-correlated in time and space. This alternative scheme should avoid the drawbacks inherent to the conventional IF method where the current-fluctuation correlator responsible of noise contains a time and space dependence coming from carrier dynamics.

In Sec. 2.1 we provided a rigorous construction of the velocity fluctuation scheme by investigating analytically the main properties of the corresponding noise source at a kinetic level using the Boltzmann-Langevin (BL) approach. However, being at a kinetic level, the BL approach involves all states in the quasi-momentum $\mathbf{p}$ space. Thus, for a real device it is practically impossible to calculate the Green-functions of the response to an initial perturbation, since they must depend on the initial and final states of a carrier in both real and momentum space. Furthermore, to obtain the two-point noise source for the conventional IF method it is necessary to calculate the cross-correlation function of velocity fluctuations under the condition of static (i.e. non-fluctuating) electric field. Consequently, to this purpose we had to finally make use of the MC approach as applied to the whole structure, thus washing out the advantages offered by simpler analytical approaches.

The objective of this section is to construct at a HD level a scheme analogous to the BL approach, thus allowing to combine the advantages associated with the physical and mathematical rigor of the BL procedure with the reasonable simplicity and good accuracy of the numerical modeling typical of HD deterministic procedures [32, 34–38, 102–109]. For this sake, similarly to the case of the BL approach, the proper microscopic noise sources must be placed into the medium equations rather than into the field equation as it is done in the conventional IF method. These medium equations will describe at the HD level the evolution of a given number of moments of the distribution function. In so doing, the velocity fluctuation scheme is here replaced by the acceleration fluctuation scheme, as firstly introduced in a heuristic way in Ref. [38].

To simply calculate the steady-state profiles of macroscopic quantities like carrier con-

centration, velocity etc. (see, for example, Figs. 2-4) one usually uses HD approaches based on conservation equations for some limited number of moments of the carrier distribution function, thus avoiding a solution of the full kinetic problem in momentum space. For transport problems such an approach has proven to be rather fruitful, providing results which are usually in a good qualitative and quantitative agreement with the results of a direct simulation of the same task at the kinetic level [35, 37, 102, 105–109]. Therefore, we believe that the generalization of the HD approach to noise problems will open new and promising perspectives.

A. Hydrodynamic Langevin forces

In the framework of the BTE, the properties of a carrier ensemble are described in terms of the single particle distribution function, $f(\mathbf{p}, x, t)$, which gives the ensemble average occupation number in phase space, here for simplicity taken three dimensional in momenta and one-dimensional in coordinates. Within such an approach, the scattering events randomly undergone by carriers, i.e. the BL forces $\xi(\mathbf{p}, x, t)$, determine the fluctuations of the distribution function around its steady-state value. Since any scattering event implies an instantaneous variation of carrier momentum but not of its coordinate, the BL force does not create directly fluctuations in the coordinate space. Therefore, an equivalent of the BL force in the HD approach can not be formulated in the same terms as in the BTE approach (i.e. in terms of the fluctuations of the occupation number), since from its nature the HD approach describes only a local balance of average characteristics of carriers, such as concentration, velocity, energy, etc., which are already quantities integrated over the whole momentum space. In other words, the description and the main features of the Langevin forces at a HD level must be formulated in terms which differ from those of the BTE level. The origin of fluctuations of the carrier characteristics described by the HD equations must directly follow from the physical source of fluctuations, i.e. from random scattering events. To obtain a comprehensive description of these sources of fluctuations at a HD level, we

apply to the BL approach the same procedure which in the non-fluctuating case allows one to go from the BTE to the HD description.

Let us start again from the BTE with the stochastic Langevin force $\xi(\mathbf{p}, x, t)$:

$$\left[\frac{\partial}{\partial t} + v(\mathbf{p})\frac{\partial}{\partial x} + eE(x,t)\frac{\partial}{\partial p_x} - \hat{S}\right] f(\mathbf{p}, x, t) = \xi(\mathbf{p}, x, t) \tag{59}$$

where $f(\mathbf{p}, x, t)$ is the single carrier distribution function, for convenience normalized to the local carrier concentration as: $n(x,t) = \int f(\mathbf{p}, x, t)d\mathbf{p}$, $v(\mathbf{p})$ the carrier group velocity, e the unit charge, $E(x,t)$ the local electric field, and $\hat{S}$ the collisional operator. Let us consider, for simplicity, electrons in the conduction band, in the absence of any generation-recombination process. The collisional term $\hat{S}[f]$ in this case describes all transitions of an electron from state $\mathbf{p}$ into another state $\mathbf{p}'$ due to all scattering mechanisms treated as acting independently.

Let the quantity $\alpha(\mathbf{p})$ characterize a certain dynamical variable of a carrier which is function of $\mathbf{p}$ only. For example, it can be the velocity $v(\mathbf{p})$, the energy $\varepsilon(\mathbf{p})$, etc. For a carrier ensemble distributed in $(\mathbf{p}, x)$-space in accordance with $f(\mathbf{p}, x, t)$, the local average density of $\alpha(\mathbf{p})$ is described by the integral quantity:

$$<< \alpha(x,t) >>= \int \alpha(\mathbf{p}) f(\mathbf{p}, x, t) d\mathbf{p} \tag{60}$$

Formally, for a given set of functions $<< \alpha(x,t) >>$ both the HD equations and the corresponding noise sources responsible of their fluctuations can be obtained by multiplying the BL equation in Eq. (59) by $\alpha(\mathbf{p})$ and integrating over the whole momentum space as:

$$\frac{\partial << \alpha >>}{\partial t} + \frac{\partial << v\alpha >>}{\partial x} - eE << \frac{\partial \alpha}{\partial p_x} >> - << \alpha \hat{S}[f] >>= \xi_\alpha \tag{61}$$

where, in accordance with Eq. (60), the double brackets $<< ... >>$ in the l.h.s. of Eq. (61) mean integration of the corresponding quantities over the distribution function in momentum space, and the quantity in the r.h.s. is explicitly given by:

$$\xi_\alpha(x,t) = \int \alpha(\mathbf{p}) \xi(\mathbf{p}, x, t) d\mathbf{p} \tag{62}$$

The quantity $\xi_\alpha(x,t)$ can be considered as the HD analog of the BTE Langevin force $\xi(\mathbf{p},x,t)$ and, as such, it describes a local source of fluctuations of the HD quantity $<<\alpha(x,t)>>$.

Since the conservation equation (61) concerns with an integral density of the corresponding dynamic characteristic, it implies the presence of many carriers in a small neighborhood of some point x, i.e. it should include the carrier concentration $n(x,t)$. As a consequence, all carriers localized temporally in the neighborhood of point x are subjected to an action of the HD Langevin force given by Eq. (62). Therefore, due to the integration over momentum space, Eq. (62) represents the total intensity of the random force which acts on all carriers located in a small neighborhood around point x.

In the general case, Eq. (61) determines the spatio-temporal behaviour of some set of HD quantities $<<\alpha(x,t)>>$, and in each conservation equation the HD Langevin force in Eq. (62) describes "spontaneous" fluctuations of the corresponding HD variable. Since the physical source of the fluctuations of various HD variables is the same, i.e. the scattering event, the $\xi_\alpha(x,t)$ and $\xi_\beta(x,t)$ which describe fluctuations of two different dynamical variables (for example, velocity ($\alpha = v$) and energy ($\beta = \varepsilon$)) cannot be treated as independent quantities, thus a certain cross-correlation will in general exist between different HD Langevin forces. In other words, the HD Langevin forces can be considered as a certain representation of the physical source of microscopic fluctuations (i.e. scattering events) applied to the description of spontaneous fluctuations of various dynamical characteristics of a carrier in the ensemble. Below we shall consider the auto- and cross-correlation functions of these HD Langevin forces and their spectral densities as well as connect them with characteristics of the scattering mechanisms.

Since the integration over momentum space in Eq. (62) leaves invariant the temporal and spatial dependencies, even at the HD level the Langevin force keeps the δ-like correlation in time and space inherent to the kinetic level. The correlation function of the HD Langevin forces that describe fluctuations of integral densities of two given dynamical characteristics $\alpha(\mathbf{p})$ and $\beta(\mathbf{p})$ can be expressed through the corresponding correlation function of the BTE

Langevin force integrated over momenta as:

$$\overline{\xi_\alpha(x,t)\xi_\beta(x',t+s)}^t = \int d\mathbf{p}\int d\mathbf{p}'\alpha(\mathbf{p})\beta(\mathbf{p}')\overline{\xi(\mathbf{p},x,t)\xi(\mathbf{p}',x',t+s)}^t = \frac{1}{2}S_{\dot\alpha\dot\beta}(x)\delta(x-x')\delta(s) \tag{63}$$

where

$$S_{\dot\alpha\dot\beta}(x) = \int d\mathbf{p}\int d\mathbf{p}'\alpha(\mathbf{p})\beta(\mathbf{p}')S_\xi(\mathbf{p},\mathbf{p}',x) \tag{64}$$

is the local spectral density of the fluctuations of the HD Langevin forces with the auto- ($\alpha = \beta$) and cross-correlation ($\alpha \neq \beta$) components expressed through the corresponding spectral density of the BL force $S_\xi(\mathbf{p},\mathbf{p}',x)$. For the general case of a collisional term which takes into account carrier scatterings with static and dynamical imperfections of the lattice as well as electron-electron (e-e) interaction, the $S_\xi(\mathbf{p},\mathbf{p}',x)$ can be given the form [110]:

$$S_\xi(\mathbf{p},\mathbf{p}',x) = 2\int d\tilde{\mathbf{p}}\int d\tilde{\mathbf{p}}' f_s(\tilde{\mathbf{p}}',x)W(\tilde{\mathbf{p}},\tilde{\mathbf{p}}')[\delta(\mathbf{p}-\tilde{\mathbf{p}})-\delta(\mathbf{p}-\tilde{\mathbf{p}}')][\delta(\mathbf{p}'-\tilde{\mathbf{p}})-\delta(\mathbf{p}'-\tilde{\mathbf{p}}')]+$$
$$\int d\tilde{\mathbf{p}}\int d\tilde{\mathbf{p}}'\int d\tilde{\tilde{\mathbf{p}}}\int d\tilde{\tilde{\mathbf{p}}}' f_s(\tilde{\mathbf{p}}',x)f_s(\tilde{\tilde{\mathbf{p}}}',x)W_{ee}(\tilde{\mathbf{p}},\tilde{\tilde{\mathbf{p}}};\tilde{\mathbf{p}}',\tilde{\tilde{\mathbf{p}}}')[\delta(\mathbf{p}-\tilde{\mathbf{p}})+\delta(\mathbf{p}-\tilde{\tilde{\mathbf{p}}})-\delta(\mathbf{p}-\tilde{\mathbf{p}}')-\delta(\mathbf{p}-\tilde{\tilde{\mathbf{p}}}')]$$
$$\times[\delta(\mathbf{p}'-\tilde{\mathbf{p}})+\delta(\mathbf{p}'-\tilde{\tilde{\mathbf{p}}})-\delta(\mathbf{p}'-\tilde{\mathbf{p}}')-\delta(\mathbf{p}'-\tilde{\tilde{\mathbf{p}}}')] \tag{65}$$

Here $f_s(\mathbf{p},x)$ is the stationary distribution function normalized to the local concentration, $\int f_s(\mathbf{p},x)d\mathbf{p} = n_s(x)$, $W(\tilde{\mathbf{p}},\tilde{\mathbf{p}}')$ is the transition rate for a carrier going from state $\tilde{\mathbf{p}}'$ into state $\tilde{\mathbf{p}}$ due to scatterings with dynamical (phonons) and static (impurities) lattice imperfections, $W_{ee}(\tilde{\mathbf{p}},\tilde{\tilde{\mathbf{p}}};\tilde{\mathbf{p}}',\tilde{\tilde{\mathbf{p}}}')$ is the corresponding transition rate for e-e interaction when two electrons with initial states $\tilde{\mathbf{p}}'$ and $\tilde{\tilde{\mathbf{p}}}'$ are scattered into final states $\tilde{\mathbf{p}}$ and $\tilde{\tilde{\mathbf{p}}}$. By substituting Eq. (65) into Eq. (64) and performing the integration over the final states one obtains the following expression for the spectral density of the HD Langevin forces:

$$S_{\dot\alpha\dot\beta}(x) = 2\int d\tilde{\mathbf{p}}\int d\tilde{\mathbf{p}}'[\alpha(\tilde{\mathbf{p}})-\alpha(\tilde{\mathbf{p}}')][\beta(\tilde{\mathbf{p}})-\beta(\tilde{\mathbf{p}}')]W(\tilde{\mathbf{p}},\tilde{\mathbf{p}}')f_s(\tilde{\mathbf{p}}',x)$$
$$+\int d\tilde{\mathbf{p}}\int d\tilde{\mathbf{p}}'\int d\tilde{\tilde{\mathbf{p}}}\int d\tilde{\tilde{\mathbf{p}}}'[\alpha(\tilde{\mathbf{p}})+\alpha(\tilde{\tilde{\mathbf{p}}})-\alpha(\tilde{\mathbf{p}}')-\alpha(\tilde{\tilde{\mathbf{p}}}')][\beta(\tilde{\mathbf{p}})+\beta(\tilde{\tilde{\mathbf{p}}})-\beta(\tilde{\mathbf{p}}')-\beta(\tilde{\tilde{\mathbf{p}}}')]$$
$$\times W_{ee}(\tilde{\mathbf{p}},\tilde{\tilde{\mathbf{p}}};\tilde{\mathbf{p}}',\tilde{\tilde{\mathbf{p}}}')f_s(\tilde{\mathbf{p}}',x)f_s(\tilde{\tilde{\mathbf{p}}}',x) \tag{66}$$

As follows from Eq. (66), $S_{\dot\alpha\dot\beta}(x)$ is determined only by processes occurring during a single scattering event through changes of the dynamical variables of the particles involved in

scatterings. In the case of a scattering with static or dynamic lattice imperfections (first term in the r.h.s. of Eq. (66)), $S_{\dot{\alpha}\dot{\beta}}(x)$ are determined by the rates of instantaneous variations of dynamical variables of a single carrier, $\Delta_\alpha = \alpha(\tilde{\mathbf{p}}) - \alpha(\tilde{\mathbf{p}}')$, such as velocity ($\alpha = v$), energy ($\alpha = \varepsilon$), etc.; these variations take place when the carrier make a transition from state $\tilde{\mathbf{p}}'$ into state $\tilde{\mathbf{p}}$ due to an interaction event. In the case of e-e interaction, the contribution into $S_{\dot{\alpha}\dot{\beta}}(x)$ comes from variations of the net dynamical variables of the two-particle subsystem. It can be, for example, the variation of the net velocity or energy which the two particles undergo during their collision. However, it should be stressed that the energy and momentum variations of two particles ungergoing an e-e scattering satisfy the conservation laws, i.e. they are strongly correlated. When such a correlation does not imply variations of the net dynamical variables during the scattering event, this last does not contribute to the spectral density of HD Langevin forces. Therefore, for a parabolic single-valley band model e-e scattering does not contribute to fluctuations at the HD level and thus the second term in the r.h.s. of Eq. (66) is equal to zero. Of course, an indirect influence of the e-e scattering on $S_{\dot{\alpha}\dot{\beta}}(x)$ can exist through the stationary distribution function entering the single-particle scattering term (the first term in the r.h.s. of Eq. (66)). A certain contribution of e-e scattering into $S_{\dot{v}\dot{v}}(x)$ can be expected only in the case of non-parabolic or anisotropic band structures. In any case, this contribution can be neglected when compared with that of carrier scatterings due to lattice imperfections (the first term in Eq. (66)). Therefore, in the following we shall disregard e-e scattering.

It should be underlined, that by substituting $\alpha = 1$ into Eq. (66) one obtains that the spectral density of the corresponding Langevin force is equal to zero in the whole frequency range, and that any cross-correlation of this force with other HD Langevin forces is absent, i.e. if $\alpha = 1$, then $S_{\dot{\alpha}\dot{\beta}} = 0$ for any β. Since in this case Eq. (61) for $\alpha = 1$ gives the balance equation for carrier concentration, it means, that there is no HD Langevin source for the fluctuations of concentration and thus, in the absence of generation-recombination processes, any fluctuation of concentration should be induced by other HD sources. As a consequence, the HD Langevin forces given by Eq. (62) describe fluctuations only of that

dynamical variables which can change during a scattering event.

The variations of a dynamical variable which determines the spectral density of the HD Langevin forces can be represented in terms of rates of variations (henceforth called *accelerations* to emphasize the difference with the velocity fluctuation scheme of Sec. 2) of the dynamical variable caused by each scattering event. As shown in Ref. [38], within the framework of the stochastic interpretation of the BTE in terms of a random walk of a single carrier in momentum space, the spectral densities of the HD Langevin forces are determined by the auto-correlation function of accelerations undergone by the dynamical variables during single scattering events. Furthermore, the cross-correlation function of these accelerations describes the relaxation processes which are usually accounted for both in the BTE approach and HD equations by the collisional and relaxation terms, respectively. Such an interpretation allows one to introduce the concept of the spectral density of Langevin forces associated with the single-particle, $\tilde{S}_{\dot{\alpha}\dot{\beta}}$. This quantity is determined over a trajectory of the single-carrier random walk in momentum space as [38]:

$$\tilde{S}_{\dot{\alpha}\dot{\beta}} = \frac{2}{T}\sum_{i=1}^{N}\Delta\alpha_i\Delta\beta_i \tag{67}$$

where $\Delta\alpha_i, \Delta\beta_i$ are the instantaneous variations of some dynamical variables of a single carrier during the i-th scattering event and T is the total time elapsed during N scattering events.

The spectral density of the HD Langevin forces given by Eq. (66), and which comes from the BL approach, can be related to the single-particle representation given by Eq. (67) in a quite natural way. Indeed, as follows from Eq. (66) the power of the HD noise source is distributed in the volume proportionally to the steady-state carrier concentration $n_s(x_0) = \int f_s(\mathbf{p}, x_0)d\mathbf{p}$. Accordingly, hot-carrier effects are accounted for by the local distribution of carriers in the momentum space at the steady-state $f_s(\mathbf{p}, x_0)/n_s(x_0)$. In order to introduce the single-particle spectral densities it is necessary to separate concentration and hot-carrier effects by factorizing the spectral density given by Eq. (66) as:

$$\tilde{S}_{\dot{\alpha}\dot{\beta}}(x_0) = S_{\dot{\alpha}\dot{\beta}}(x_0)/n_s(x_0) \tag{68}$$

In Eq. (68) the local values directly depend on the single-carrier distribution function in the whole momentum space and in the neighborhood of point $x = x_0$. Since for most mechanisms the scattering rate depends only on carrier energy, it is reasonable to assume that the $\tilde{S}_{\dot{\alpha}\dot{\beta}}$ depends only on the local mean energy, $< \varepsilon >=<< \varepsilon(x) >> /n_s(x)$. In this case numerical values of $\tilde{S}_{\dot{\alpha}\dot{\beta}}(< \varepsilon >)$, as functions of $< \varepsilon >$ can be obtained from MC simulations of the homogeneous bulk material in accordance with Eq. (67).

As follows from the above considerations, in going from the BTE to the HD level of description, the properties of the fluctuation source, described in both cases by the Langevin force, must be reformulated. In the BTE approach the source is represented by fluctuations of the occupation number, i.e. the number of particles in a certain volume of momentum space in the neighborhood of some point x, due to scattering events. In the HD approach the source of fluctuations is represented by accelerations of carrier dynamical characteristics associated with interaction events with the thermal bath or with other carriers. By factorizing the spectral density of the HD Langevin forces into the product of carrier concentration and single-particle spectral density, the microscopic part of the problem is reduced to the determination of the local microscopic noise sources. This task can be accomplished by performing a kinetic modeling of the bulk material.

As an example, Fig. 12 shows the HD noise sources associated with velocity and energy fluctuations and their cross-correlation as functions of the excess mean energy $< \varepsilon > -\varepsilon_{th}$, ε_{th} being the thermal equilibrium value, calculated with MC simulations for bulk n-type GaAs at room temperature. The diagonal sources $S_{\dot{v}\dot{v}}(< \varepsilon >)$ and $S_{\dot{\varepsilon}\dot{\varepsilon}}(< \varepsilon >)$ start from their thermal equilibrium values and then monotonously increase with increasing excess mean energy. This increase reflects both the broadening of the carrier distribution in momentum space and the increase with energy of the total scattering rate. Furthermore, one can detect two different slopes at relatively low ($< \varepsilon > \ < 300\ meV$) and high ($< \varepsilon > \ > 300\ meV$) energy values, which correspond to the fact that most of electrons are in the lower and upper valleys, respectively.

The velocity-energy cross-correlation term, being an odd momentum of the distribution,

under thermal equilibrium is equal to zero. This is no longer the case under hot-carrier conditions when the electric field couples velocity and energy relaxations. Indeed, during each scattering event a carrier in average loses both the extra velocity and energy gained by the electric field. Thus, the changes of velocity and energy during scattering events are negative in average, so that their average product, $S_{\dot{v}\dot{\varepsilon}}(<\varepsilon>)$, is positive. Therefore, from Fig. 12 we see that the cross-correlation term, $S_{\dot{v}\dot{\varepsilon}}(<\varepsilon>)$, starts from its zero value at thermal equilibrium, rapidly increases due to velocity-energy cross-correlation in the lower valley, and finally attains a slope determined by the relaxation processes in the upper valleys. Remarkably, in the intermediate energy range the cross-correlation term exhibits an N-shape behavior.

Below we shall illustrate the HD-Langevin scheme by presenting the results of numerical calculations performed for the same submicron GaAs structure of Sec. 2. Figure 13 shows the spatial profiles of the microscopic noise sources entering the acceleration fluctuation scheme: namely, $S_{\dot{v}\dot{v}}$, $S_{\dot{v}\dot{\varepsilon}}$ $S_{\dot{\varepsilon}\dot{\varepsilon}}$. The values are calculated by using the data of Fig. 12 and the energy profile $\varepsilon(x)$ already reported in Sec. 2 (see Fig. 2 (c)). Let us stress again the frequency independence of the microscopic noise sources. Their spatial dependence follows their energy dependence shown in Fig. 12. The magnitude of all noise sources reaches a maximum near the nn^+ anode homojunction where also the mean energy achieves its maximum values (see Fig. 2 (c)). It should be underlined, that under the same voltage drop between the structure terminals the stationary profiles of velocity, energy, concentration, etc. inside the two-terminal n^+nn^+ structure remain the same for the different conditions in which the structure can be connected with an external circuit. Therefore, these microscopic noise sources remain the same for different noise-operation modes.

B. Hydrodynamic Green-functions

Usually, it is convenient to separate the effects related to the redistribution of carrier concentration from those caused by carrier heating in the self-consistent electric field. The

latter are described in terms of average values of dynamical characteristics taken by a single carrier in the neighborhood of a given point x:

$$< \alpha(x,t) >= \frac{1}{n(x,t)} \int \alpha(\mathbf{p}) f(\mathbf{p},x,t) d\mathbf{p} = \frac{<< \alpha(x,t) >>}{n(x,t)} \tag{69}$$

where $n(x,t)$ obeys the continuity equation:

$$\frac{\partial n}{\partial t} + \frac{\partial n < v >}{\partial x} = 0 \tag{70}$$

which follows from Eq. (61) by taking $\alpha = 1$. By subtracting Eq. (70) multiplied by $< \alpha(x,t) >$ from Eq. (61) and then dividing the result by the local carrier concentration one obtains:

$$\frac{\partial}{\partial t} < \alpha > + < v > \frac{\partial}{\partial x} < \alpha > + \frac{1}{n} \frac{\partial}{\partial x} [n < \delta v \delta \alpha >] - eE < \frac{\partial \alpha}{\partial p_x} > - < \alpha \hat{S}[f] >= \frac{1}{n} \xi_\alpha \tag{71}$$

where $< \delta v \delta \alpha >=< v\alpha > - < v >< \alpha >$.

Usually, the system of Eqs. (71) is written for successive moments of the distribution function, $\alpha(p) = p^m$, where $m = 0$ gives the particle conservation equation, $m = 1$ the momentum conservation equation, etc. ¿From a mathematical point of view, the set of conservation equations containing all moments is equivalent to the BL scheme. In practical calculations, one is forced to truncate the system to a limited number of equations. In so doing, the conservation equation for a moment of given order, $< \alpha >$, always involves $< \delta v \delta \alpha >$ which depends on the higher order moment $< v\alpha >$. Therefore, to close the system at a given order it is necessary to make some assumptions for the higher-order moment. ¿From the physical point of view, the most important variables are the velocity and energy of the single carrier. Thus, a reasonable truncation of the system can be made at the second-order moment. To this purpose, in accordance with Refs. [101, 102] we make the two following assumptions: (i) that $< \delta v \delta \alpha >$ depends only on the local instantaneous mean energy and coincides with the value of $Q_\alpha =< \delta v \delta \alpha >_0$ calculated with some kinetic approach for the homogeneous case at the same mean energy; (ii) that the collisional term is given by the relaxation time approximation as: $< \alpha \hat{S}[f] >= -(< \alpha > -\alpha_{th})\nu_\alpha$ where ν_α is the relaxation rate of the quantity $< \alpha >$ to its thermal equilibrium value α_{th}.

Within the above assumptions, the l.h.s. of the conservation equations for $v(x,t)$ and $\varepsilon(x,t)$ coincides with Eqs. (51)-(52) of Sec. 2:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{1}{n}\frac{\partial}{\partial x}(nQ_v) - eEm^{-1} + v\nu_v = \frac{1}{n}\xi_v \tag{72}$$

$$\frac{\partial \varepsilon}{\partial t} + v\frac{\partial \varepsilon}{\partial x} + \frac{1}{n}\frac{\partial}{\partial x}(nQ_\varepsilon) - eEv + (\varepsilon - \varepsilon_{th})\nu_\varepsilon = \frac{1}{n}\xi_\varepsilon \tag{73}$$

where, for simplicity of notation, averaging brackets are omitted.

Without the Langevin forces this set of HD equations has been already proved to describe satisfactorily both static and dynamic characteristics of semiconductor structures, even when the characteristic lengths of spatial variations of main HD variables in the active region of a structure are comparable with the carrier mean free-path [35, 37, 102].

In the framework of the single-carrier representation of the HD equations, the factor $1/n$ in the r.h.s. of Eqs. (72) and (73) normalizes to a single particle the action of the Langevin force given by Eq. (62). Since in the following we shall be interested only in linear fluctuations caused by HD Langevin forces under stationary conditions, the carrier concentration $n(x,t)$ in the r.h.s. of Eqs. (72) and (73) must be replaced by its stationary non-fluctuating value $n_s(x)$ when constructing the response functions (Green functions) caused by the local perturbation $\delta(x-x_0)\delta(t)/n_s(x_0)$.

Let us consider the general case of a numerical calculation of the Green-function (response function) for an arbitrary local characteristic of the system, $H(n,v,\varepsilon,E,x)$, which is represented as a function of the HD variables (n,v,ε), the local self-consistent electric field E and position x inside the structure. Examples of such an H-characteristic are the HD variables themselves, the electric field, the conduction current, $j_{cond} = env$, the energy flux, $W = env\varepsilon$, etc. The general procedure for the calculation of the Green functions is quite similar to that described in Sec. 2 for the calculation of the conventional IF. Firstly, in the absence of perturbations due to the Langevin forces, one should obtain a stationary solution of Eqs. (70), (72) and (73) coupled with the Poisson equation and, if necessary, another equations describing the external circuit (see Sec. 3.3). Then, a perturbation of the steady-state values of velocity or energy, given by $\Delta_\alpha\delta(x-x_0)$, is introduced at time $t=0$

and point x_0. Usually, the spatial profile of the perturbation is given by some approximation of the δ-function which takes some volume in x-space, as for example a Gaussian function (see Eq. 56). The perturbation amplitude Δ_α is taken sufficiently small to fulfill the requirement of linearity of the response. Then, a direct numerical solution of the system of equations (70), (72) and (73) jointly, if necessary, with the Poisson and circuit equations is carried out to give the relaxation of the system to the stationary state. The Green-function corresponding to the given H-characteristic is obtained from the difference between the local values of $H(x, x_0, t)$ calculated during the relaxation process and the values of $H_s(x)$ corresponding to steady-state conditions. The values of the difference are then normalized to the amplitude of the initial perturbation Δ_α and the carrier concentration $n_s(x_0)$ at the point of the perturbation as:

$$G_H^\alpha(x, x_0, t) = \frac{1}{\Delta_\alpha n_s(x_0)}[H(x, x_0, t) - H_s(x)] \tag{74}$$

where $\alpha = v, \varepsilon$, respectively.

In accordance with the Green-function formalism, the local fluctuation of an H-characteristic caused by the HD Langevin force (62) can be represented as:

$$\delta H(x, t) = \sum_{\alpha=v,\varepsilon} \int dx_0 \int_0^\infty d\tau G_H^\alpha(x, x_0, \tau)\xi_\alpha(x_0, t - \tau) \tag{75}$$

Let us note, that in accordance with Eq. (75) the factor $1/n_s(x)$, which normalizes the action of the HD Langevin force $\xi_\alpha(x_0, t)$ to a single particle, is separated from the force itself and placed into the definition of the Green function $G_H^\alpha(x, x_0, \tau)$ given by Eq. (74). This allows us to consider $G_H^\alpha(x, x_0, \tau)$ as a single-particle Green-function which describes the linear response of the H-characteristic to a local perturbation of the dynamical variable of a single particle. The dependence of the total response $\delta H(x, t)$ in Eq. (75) on $n_s(x)$ is contained in $\xi_\alpha(x_0, t)$, whose spectral power $S_{\dot\alpha\dot\beta}(x_0)$ is proportional to $n_s(x)$, as it was shown in Sec. 3.1. Owing to the δ-like correlation of the Langevin forces both in time and space (see Eq. (63)), the spatio-temporal dependence of the auto-correlation ($H = H'$) and cross-correlation ($H \neq H'$) functions of fluctuations of any two H-characteristics takes the

form:

$$C_{HH'}(x', x'', s) = \overline{\delta H(x', t)\delta H'(x'', t-s)}^{t} =$$

$$= \frac{1}{2} \sum_{\alpha,\beta=v,\varepsilon} \int dx_0 n_s(x_0) \tilde{S}_{\dot{\alpha}\dot{\beta}}(x_0) \int_0^{\infty} G_H^{\alpha}(x', x_0, \tau) G_{H'}^{\beta}(x'', x_0, \tau + s) d\tau \qquad (76)$$

¿From a formal point of view, the procedure of calculation of any HD Green-function proposed above allows one to analyse the further spatio-temporal evolution of the perturbation of any local characteristic of the system. Furthermore, the representation in terms of the correlation function of fluctuations given by Eq. (76) forms the background for any calculation of electronic noise in one-dimensional structures described by the HD equations.

Let us illustrate the HD-Langevin approach developed above by numerical calculations of the linear response functions of some characteristics of the system and of the spatio-temporal correlation functions of their fluctuations. These calculations will also allow us to show how to obtain the spatially-correlated noise source of Sec. 2 by a direct HD calculation under static electric field conditions. The requirement of a static electric field is realized for the system of Eqs. (70), (72) and (73) in a natural way by replacing $E(x,t)$ in Eqs. (72) and (73) by $E_s(x)$, thus providing the closure of the HD system without incorporating additional equations for the self-consistent electric field and the external circuit.

Figures 14 shows the spatio-temporal evolution of the response of carrier velocity, concentration, and conduction current (i.e. the Green functions of the corresponding H-characteristics) calculated under static electric-field conditions for an initial velocity perturbation placed in point $x_0 = 0.315\ \mu m$ of a $0.21 - 0.3 - 0.39\ \mu m$ n^+nn^+ GaAs structure with dopings $n = 5 \times 10^{15}$ and $n^+ = 10^{17}\ cm^{-3}$ at $T = 300\ K$ and $U_d = 0.5\ V$, i.e. the same submicron structure of Sec. 2. For the HD modeling the whole structure was subdivided into 900 meshes with uniform spatial step of 1 nm. The initial perturbation in space was approximated by a narrow Gaussian distribution with a semi-width corresponding to 2 meshes. The electron flux is taken to move from the left contact (cathode) to the right one (anode).

Figure 14(a) shows the evolution of the velocity profile. The initial perturbation of veloc-

ity given at $t = 0$ as a narrow Gaussian firstly reduces in magnitude suffering some diffusive broadening, and then quickly splits into two peaks which start to propagate separately along the structure. Due to both the inhomogeneity of the structure and the presence of a current flux, the further evolution of perturbation is rather asymmetric in space, especially in the forward and backward directions, with respect to the carrier flux. In the general case the two split peaks move in opposite directions with velocities estimated as $v \pm v_D$, where v and v_D are the average drift and diffusion velocities [37]. Since at the point $x = 0.315\ \mu m$, where the perturbation is introduced, $v \approx v_D$ the left peak practically remains at the same place while the right peak quickly moves towards the anode contact.

When the velocity perturbation propagates in the structure it induces the perturbation of other characteristics such as concentration, conduction current, etc. The induced perturbations of $n(x,t)$ and $j_{cond}(x,t)$ initially take a typical dipole-domain shape. For example, Fig. 14 (b) shows the evolution of the concentration profiles. Here, the positive local perturbation of velocity quickly results in a positive perturbation of $n(x,t)$ at the right wing of the initial velocity perturbation followed (from right to left), of course, by a negative perturbation of $n(x,t)$ since the total carrier concentration in the structure remains constant. This leads to the formation of a dipole-domain shape of the $j_{cond}(x,t)$ perturbation during the first $50 \div 100\ fs$ (see Fig. 14(c)). For comparison, Fig. 14 (d) shows, in the same units as Fig. 14 (c), the part of the conduction current perturbation caused by velocity fluctuations only, $\tilde{j}(x,t)$. We notice that, due to a flat profile of the concentration, the spatial variations of $\tilde{j}(x,t)$ practically follow those of the velocity. Figures 14 (c) and 14 (d) evidence both the qualitative and quantitative differences between the spatio-temporal evolution of the induced perturbations of $\tilde{j}(x,t)$ and $j_{cond}(x,t)$.

To obtain the two-point noise source for the conventional IF method described in Sec. 2, as a local H-characteristic one must use the conduction current fluctuations caused by velocity fluctuations only, thus omitting the concentration fluctuations, i.e. $H = \tilde{j}(x,t) = en_s(x)v(x,t)$. As an example, Fig. 15 compares the cross-correlation functions of conduction current fluctuations associated with velocity fluctuations only $C(n,m,t)$

calculated under static field conditions with the MC (curves 1 and 2) and HD (curves 3 and 4) approaches, respectively, for cells m centered at points $x = 0.435,\ 0.495\ \mu m$ (curves 1,3 and 2,4, respectively) and a cell n centered at $x = 0.315\ \mu m$ in the same submicron structure of Sec. 2. The spectral densities of the HD Langevin forces, necessary to calculate the cross-correlation functions in accordance with Eq. (76), are those reported in Figs. 12 and 13. A comparative analysis of the contributions which the various noise sources give in Eq. (76) to the cross-correlation functions of velocity fluctuations for the submicron structure of Fig. 13 shows that the main contribution comes from velocity fluctuations, and that energy fluctuations are practically negligible. By substituting the full set of $C(n, m, t)$ obtained from the present HD modeling into Eq. (27) for the noise source $K(x', x'', \omega)$ and using the IF developed in Sec. 2, through Eq. (26) one can calculate the spectral density of voltage fluctuations. The result is presented in Fig. 16 together with the $S_U(\omega)$ directly evaluated by the MC simulation. To evaluate the role of the different noise sources, curve 3 in Fig. 16 shows the contribution due only to the $S_{\dot{v}\dot{v}}$-term in Eq. (76). By comparing curve 3 with curve 2, which contains all contributions, we conclude that the main contribution to the noise comes from the direct perturbations of velocity. Indeed, when an initial perturbation of energy is introduced, the induced perturbation of velocity appears only at a later time, due to a perturbation of the energy-dependent parameters in the average velocity conservation equation. Since any electric field and conduction current perturbations are induced only through velocity fluctuations, the appearance of these perturbations is less direct when promoted by energy perturbations with respect to the case when promoted by velocity perturbations. Due to this reason, the processes induced by direct energy perturbations are usually of minor importance, and the main contribution to fluctuations of macroscopic quantities comes from velocity fluctuations during scattering events.

The qualitative and quantitative agreement between the HD and MC approaches shown in Figs. 15 and 16 is considered to be satisfactory to validate the present scheme. We conclude that the HD-Langevin approach developed here allows one to calculate the spatially-correlated noise source entering the conventional IF method. In particular, in this scheme

the noise calculation from the conventional IF formula does not require a MC simulation of the corresponding structure. However, although the problem of calculating the spectral density of voltage fluctuations between the structure terminals is entirely solved in the framework of the HD approach, from the physical point of view one is facing a rather strange situation: in the velocity fluctuation scheme considered in Sec. 2 there are two kind of noise sources. Indeed, a first one are the spatially-correlated velocity fluctuations which enter as noise source of the conventional IF formula; but in order to calculate such a source it is necessary to introduce a second and more primitive noise source directly related to the accelerations which carriers undergo during scattering events. Furthermore, the same system of medium equations are used to calculate both the IF and the corresponding noise source. Since from the physical point of view the source of fluctuations is unique, that is the scattering events, the representation of the noise source in the conventional IF method in terms of a two-step procedure is a rather artificial scheme. A natural way to overcome this problem is to reformulate the conventional IF method by keeping only the primitive noise source which reflects the stochastic nature of fluctuations directly related to the accelerations caused by scattering events. In this case the spectral characteristics of the noise source are determined by scattering events only and do not include any dynamical motion of carriers between scattering events.

Since the final goal of any noise calculation is to provide the spectral densities which can be experimentally measured or used for a further analysis of the circuit noise in terms of voltage and current noise-generators, such a generalization of the conventional IF formula must be implemented in a way which allows to describe the noise and fluctuations in the system by a generalized IF while keeping the microscopic noise sources as the unique cause of fluctuations. As follows from Eqs. (74)-(76) the auto- and cross-correlation functions of various H characteristics can be calculated in a similar way, as it was done above for the two-point noise source of the conventional IF method. Therefore, by representing through H-characteristics quantities as the voltage drop between the structure terminals or the current flow through the structure, one can in principle directly calculate the spectral densities of

the fluctuations of these characteristics. For this purpose, the HD conservation equations (70), (72) and (73) must be coupled with the Poisson and the external circuit equations so that the proper operation mode can be directly simulated. This task is considered in the following section.

C. Operation modes

As simplest condition, we consider the constant current operation mode described in detail in Sec. 2. The equation for the spatio-temporal evolution of the self-consistent electric field $E(x,t)$ inside the structure (see Eq. (50)) now does not contain its own source of fluctuations in the r.h.s., and can be written as:

$$\frac{\partial E}{\partial t} + v\frac{\partial E}{\partial x} + \frac{e}{\epsilon\epsilon_0} v N_d - \frac{1}{\epsilon\epsilon_0} J = 0 \tag{77}$$

In full analogy with the IF method described in Sec. 2, to determine the carrier concentration profile, instead of Eq. (70) we shall use the Poisson equation rewritten in the form which allows us to calculate $n(x,t)$ from the knowledge of the instantaneous profile of $E(x,t)$ obtained from the solution of Eq. (77) as:

$$n(x,t) = N_d(x) + \frac{\epsilon\epsilon_0}{e}\frac{\partial E(x,t)}{\partial x} \tag{78}$$

In the framework of the HD-Langevin approach considered above, the source of fluctuations is now moved from the field equations (as it was for the conventional IF method) into the medium equations (see Eqs. (72)-(73)). As a result, it is no longer necessary to subdivide artificially velocity fluctuations into two parts as done in the conventional IF method where we introduced: (i) the stochastic part which directly plays the role of noise source, i.e. the velocity fluctuations appearing in the absence of electric field perturbations, and (ii) the transfer part which is induced by the perturbation appearing in the electric field and is responsible for the further spatio-temporal evolution of fluctuations, i.e. the transfer field. Now the primitive noise source for both parts of velocity fluctuations is the same

and is represented by the spontaneous accelerations undergone by carriers during scattering events.

Under constant current operation mode, the total current density J flowing through the structure is constant in time and must be considered as a quantity which is fixed outside the structure. Therefore, the system of Eqs. (72), (73), (77) and (78) constitutes a closed system for the calculations of fluctuations of the voltage drop between the structure terminals:

$$U_d(t) = \int_0^L E(x,t)dx \tag{79}$$

where L is the structure length.

In the framework of the Green-functions method described above, the generalized IF formula with the primitive source of fluctuations determined by Eqs. (66) and (68) is obtained when the voltage drop between the structure terminals given by Eq. (79) is used as H-characteristic, i.e. $H_U(t) \equiv U_d(t)$. Since $U_d(t)$ is represented as the H-characteristic integrated over the whole structure, the corresponding correlation function given by Eq. (76) is independent of spatial variables and describes the correlation of fluctuations of the voltage drop between the structure terminals. By applying the Wiener-Khintchine theorem, similarly as it was done in Sec. 2.2, one obtains the generalized IF formula in the form:

$$S_U(\omega) \equiv 2\int_{-\infty}^{\infty} \overline{\delta H_U(t)\delta H_U(t+s)}^t exp(i\omega s)ds =$$

$$= \sum_{\alpha,\beta=v,\varepsilon} \int_0^L dx_0 n_s(x_0)\tilde{S}_{\dot{\alpha}\dot{\beta}}(x_0)\nabla Z_{\dot{\alpha}}(x_0,\omega)\nabla Z^*_{\dot{\beta}}(x_0,\omega) \tag{80}$$

where

$$\nabla Z_{\dot{\alpha}}(x_0,\omega) = \int_0^{\infty} dsexp(-i\omega s)G_U^{\alpha}(x_0,s) \tag{81}$$

is the generalized IF determined as the Fourier transform of the function $G_U^{\alpha}(x_0,s)$ which gives the linear response of the H_U-characteristic (i.e. the voltage drop between the structure terminals) to perturbations of velocity and energy ($\alpha = v,\ \varepsilon$) appeared at point x_0.

Similarly to its conventional analogue given by Eq. (1), the generalized IF formula given by Eq. (80) represents $S_U(\omega)$ as the volume integral of a local distribution of the

power density of the source of fluctuations inside the structure which is transferred by a generalized IF, $\nabla Z_{\dot{\alpha}}(x_0, \omega)$, to the structure terminals as voltage fluctuations. However, here the similarity stays only on the formal mathematical equivalence of the representations given by the generalized and conventional IF formula. ¿From the physical point of view these representations differ substantially, since the nature of the noise source is essentially different. In the generalized IF formula the noise source is a primitive one: it is the microscopic force which describes the spontaneous accelerations undergone by carriers during scattering events. By contrast, in the conventional IF formula the noise source is represented by a complicated physical quantity corresponding to the macroscopic fluctuations of the carrier flux, which includes both spontaneous processes occurring during scatterings and carrier dynamics between scattering events. Both in the generalized and conventional formulations, the IF is described in a similar way by the response of the electric field inside the structure to an initial local perturbation introduced by the noise source. However, since the noise source in the generalized IF formula differs from its classical analogue, the generalized IF cannot be directly related to the small-signal impedance of the structure, which we recall in the conventional IF is determined as [64–66]: $Z_d(\omega) = \int_0^L \nabla Z(x_0, \omega) dx_0$. Nevertheless, we shall keep the usual notation ∇Z for the generalized IF too, by adding subindex $\dot{\alpha}$ where it is necessary to stress the difference.

The representation of a spectral density given by Eq. (80) in terms of a spatial convolution of the density of the source power and corresponding transfer functions (the generalized IF) is not restricted to the case of constant current operation mode. Such a representation can be extended to the general case of a structure operation mode in an arbitrary external circuit. For this purpose, it is necessary to add to Eqs. (72), (73), (77) and (78) the equations relating the voltage drop between the structure terminals $U_d(t)$ and the current flow through the structure $J(t)$ with the parameters of the external circuit, as it was done, for example, in Refs. [11, 102, 111]. Here we shall discuss the simplest example of an external circuit, when the structure is connected in series with a load non-inertial resistance R and a certain voltage supply. Let us suppose that: (i) both the load resistance and the

source of voltage supply U applied to the whole circuit are ideal and do not contain internal sources of noise, (ii) the internal resistance of the voltage supply is already included into the magnitude of the load resistance R. In such a circuit, the voltage drop between the structure terminals $U_d(t)$ and the total current flowing through the circuit $J(t)$ are related by the following circuit equation:

$$RJ(t) + U_d(t) = U \tag{82}$$

By substituting $J(t)$ given by Eq. (82) into Eq. (77) one obtains again a closed system of field and medium equations. The numerical solution of this system allows one to calculate the electric field response inside the structure, and, hence, to represent the spectral density of voltage drop fluctuations between the structure terminals, $S_U(\omega)$, in accordance with Eqs. (80) and (81). However, while keeping the same primitive noise source, the generalized IF in Eq. (80) will depend on the structure operation mode in the external circuit described by Eq. (82). It is evident, that the character of the spatio-temporal evolution of the electric field perturbations, $E(x,t)$, and hence, the generalized transfer and IFs, will depend on the parameters of the external circuit, R in the considered case. As follows from Eq. (82), there exist two main ways of connecting the structure to an external circuit when the spectral density of fluctuations is independent of the external circuit parameters. They correspond to the two limiting cases when $R \to \infty$ or $R \to 0$.

In the first case the external circuit damps entirely any fluctuations of the current, $\Delta J = \Delta U_d/R \to 0$, and only the voltage drop between the structure terminals can fluctuate. A trivial situation is the limiting case of an open circuit with $U/R \to 0$, when any current is absent in the external circuit, i.e. $J = 0$. Here only Nyquist fluctuations of the voltage drop, corresponding to thermal equilibrium [65], are obtained. In the case of an infinitely large power of the voltage supply, when U and R simultaneously tend to very large values while keeping the ratio $U/R \to J_0 = const$ constant, the operation mode of constant total current $J_0 = const$, is achieved. In this case, the fluctuations of voltage drop ΔU_d appearing between the structure terminals will correspond to the non-equilibrium conditions created by

the ideal non-fluctuating current flux through the structure J_0. Their spectral density $S_U(\omega)$ describes the so called Norton source of fluctuations, i.e the voltage noise generator which is usually introduced within a phenomenological description of electrical noise in circuits.

The second operation mode, when the fluctuations appearing in the external circuit are independent of circuit parameters, can be realized under the condition that voltage fluctuations between the structure terminals are completely damped by the circuit, i.e. $\Delta U_d = 0$. This operation mode is realized when $R \to 0$, so that the whole voltage given by the voltage supply is applied to the structure, i.e. $U_d = U$ (the so called constant applied-voltage operation). In this case the only fluctuating quantity is the current flowing through the structure, J, and the spectral density of current fluctuations, $S_J(\omega)$, describes the so called Thevenin source of fluctuations, i.e. the current noise generator which is usually introduced within a phenomenological description of electrical noise in circuits.

It is well known that for two-terminal devices, the spectral densities of the Norton and Thevenin noise generators are mutually related through the small-signal impedance of the structure, $Z_d(\omega)$ as [65]:

$$S_J(\omega) = \frac{S_U(\omega)}{|Z_d(\omega)|^2} \tag{83}$$

Let us show that under the constant applied-voltage operation mode $S_J(\omega)$ also can be directly calculated in the framework of the HD-Langevin scheme and it can be described by expressions similar to Eqs. (80) and (81). This possibility is achieved by introducing the response functions which directly describe the fluctuations of the total current in the external circuit initiated by single scattering events (i.e. the primitive noise source). Analogously to the case of constant current operation, such a representation shall be called *generalized admittance field (AF) formula.* ¿From a formal point of view, there appear some problems when trying to carry out calculations for such an operation mode by using Eq. (78) which directly describes the spatio-temporal evolution of the self-consistent electric field inside the structure. The requirement of the absence of any time-variations of the voltage drop between

the structure terminals, i.e.

$$\frac{dU_d}{dt} \equiv \int_0^L \frac{\partial E(x,t)}{\partial t} dx = 0 \tag{84}$$

means that the displacement currents $j_{disp} \sim \partial E(x,t)/\partial t$ caused by local time-variations of the electric field must be entirely self-compensated inside the structure. With this requirement to be satisfied, the dynamical Eq. (77) transforms into a trivial relation between the total current and the conduction current component:

$$J = \frac{e}{L} \int_0^L n(x,t) v(x,t) dx \tag{85}$$

which can not be used for the description of a spatio-temporal evolution.

These difficulties related to the representation based on Eq. (77) can be overcome by using an alternative representation based on the direct description of the spatio-temporal evolution of the carrier concentration determined by the charge conservation equation given in our case by the continuity Eq. (70). In this case, the $E(x,t)$, which follows the redistribution of $n(x,t)$ will be determined by a direct solution of the Poisson equation given by Eq. (78) with U_d determined by the external circuit. Such a solution can be represented as the sum of two terms:

$$E(x,t) = E_\rho(x,t) + E_b(t) \tag{86}$$

where

$$E_\rho(x,t) = \frac{e}{\epsilon\epsilon_0} \int_0^x [n(x',t) - N_d(x')] dx' \tag{87}$$

is the electric field component directly related to the space-charge distribution inside the structure, and

$$E_b(t) = \frac{1}{L} \left[U_d(t) - \int_0^L E_\rho(x,t) dx \right] \tag{88}$$

is the electric field component governed by the external circuit, i.e. by the voltage drop $U_d(t)$. As follows from Eq. (88), this component covers spatially the whole structure, thus reflecting the appearance of a global feedback which determines the spatio-temporal evolution of perturbations through the external circuit. This feedback transforms a local fluctuation gradually propagating through spatially-coupled regions (see, e.g. [37]) into a

global fluctuation which covers at once the whole structure. Let us stress once more, that this feedback appears only in the presence of some restrictions on voltage fluctuations: for example, by using Eq. (82) for the circuit operation or Eqs. (86)-(88) for the case of constant voltage applied between the structure terminals. For the ideal constant current operation mode, when the spatio-temporal behavior of the electric field is described by Eq. (77), such a feedback is absent and the spatio-temporal evolution of fluctuations keeps a local character of propagation through spatially-coupled regions (see also Appendix for more details).

We conclude that Eqs. (70), (72), (73), (78) and (86) form a closed system which allows us to describe the fluctuations under the operation mode of constant applied voltage. In this case, to calculate the spectral density of the Thevenin generator it is suitable the use of Eq. (85) as H-characteristic representing the total current as a function of carrier concentration and drift velocity inside the structure, i.e. $H_J = J(v, n)$. In full analogy with the constant current operation mode, the spectral density $S_J(\omega)$ can be represented in terms of Eq. (80) by replacing subindex U by J and Z by Y. Furthermore, $G_J^{\alpha}(x_0, s)$ corresponds now to the time response of the H_J-characteristic and its Fourier transform $\nabla Y_{\dot{\alpha}}(x_0, \omega)$ given by Eq. (81) is the generalized AF.

The main systems of equations which have been formulated in this section together with Eqs. (80) and (81) formally close the problem of the HD-Langevin calculations of the electronic noise in the framework of the acceleration fluctuation scheme. To calculate the spectral densities of the voltage and/or current fluctuations in accordance with this general scheme one must proceed as follows:

(i) calculate the spectral density of the Langevin noise sources

(ii) simulate the spatio-temporal evolution of fluctuations initiated by the primitive noise sources,

(iii) calculate the linear response functions of voltage and/or current corresponding to the primitive perturbations,

(iv) calculate the generalized transfer fields in the spectral representation given by Eq. (81),

(v) perform the spatial integration of the noise sources and transfer fields in accordance with Eq. (80) thus obtaining finally $S_U(\omega)$ and/or $S_J(\omega)$.

In next section we shall illustrate these main steps of the general procedure by performing numerical calculations for the same submicron GaAs structure considered in the above Sec. 3.2 and in Sec. 2. The first step was already solved in Sec. 3.1. ¿From the physical point of view it is evident that the second step is the most important, since the information obtained during steps (iii)-(v) is practically a consequence of the spatio-temporal evolution of a single perturbation. To shed more light in the microscopic interpretation of the problem, in the Appendix we present an analytical formulation of the peculiarities of the spatio-temporal evolution of single fluctuations initiated by the primitive noise source in a non-homogeneous structure under various operation modes.

D. Numerical results

In the following we report numerical calculations of the noise features for the same submicron n^+nn^+ GaAs structure of Sec. 3.2 (see also Sec. 2) when operating under various operation modes. Since, as seen in Sec. 3.2, the main contribution to noise comes from instantaneous velocity accelerations, main attention will be paid to this kind of perturbation.

Figures 17 and 18 present the spatio-temporal evolution on the picosecond time scale of perturbations calculated under constant current and constant voltage operation modes, respectively. The perturbation is initiated by an instantaneous velocity acceleration placed at point $x_0 = 0.315\ \mu m$ of the structure. Analogously with the case of Fig. 14, the perturbations of the velocity and conduction current appear at once with the initial perturbation, while for an initial formation of the electric field perturbation a time interval of about $t \leq 1\ fs$ is necessary. As follows from Figs. 17 and 18, the evolution of fluctuations of the electric field and current density in the neighborhood of the point where the initial perturbation is introduced exhibits a similar behavior in both operation modes. During its evolution, the perturbation covers a larger and larger space around the point where the initial

perturbation was introduced. An essential difference in the evolution for the two different operation modes appears in spatial regions of the structure which have no direct connection with the point where the perturbation was initiated. Under constant current conditions the perturbation in these regions appears after some delay time, which is necessary for the perturbation to direct propagate far from the initial perturbation point. By contrast, under constant applied voltage, the perturbation takes at once a global character covering the whole structure. As it is shown in the Appendix, such a difference in the evolution character of the fluctuation is caused by the necessary presence of an additional feedback component of the electric field perturbation, which is responsible for the influence of the external circuit on the fluctuation evolution inside the structure when the perturbation of the total current flowing in the external circuit $\Delta J(t) = \Delta j_{cond}(x,t) + \Delta j_{disp}(x,t) \neq 0$. When $\Delta J = 0$ we have only the drift-diffusive propagation of fluctuations which occurs under conditions of local complete compensation between perturbations of the conduction and displacement currents (see the first term in Eq. (A19) and (A20)). As seen in Figs. 17 and 18, for both operation modes such a drift-diffusive behavior of the perturbation evolution always takes place in the nearest regions surrounding the point of the initial perturbation. As follows from the conservation law of total current, when $\Delta J \neq 0$ the fluctuations of the conduction and displacement currents must be synchronized in the whole structure at once. Such a spatial synchronization is achieved by the excitation of an additional (with respect to the case $\Delta J = 0$) component of the electric field perturbation (see Eq. (A13)), whose spatial profile is determined by the differential impedance of the structure (see Eq. (A18)). Under constant voltage operation mode, this component fully compensates the fluctuations of the voltage drop between the structure terminals.

The temporal and spectral behaviors of response functions of voltage and total current calculated under constant current, constant voltage and circuit operation modes are compared in Figs. 19 to 21. In accordance with Eq. (74), all response functions are normalized to the quantity $\Delta_v n_s(x_0)$. Curves 1 to 4 correspond to initial perturbations of velocity placed at points $x_0 = 0.105,\ 0.225,\ 0.315,\ 0.435\ \mu m$ of the same submicron structure of

Fig. 13. As follows from Fig. 19 (a), under constant current operation mode all curves for the voltage response function starts from zero, show some time dependence and finally go to zero again when any perturbations inside the structure vanish. On the other hand, under constant voltage operation mode, since the average value of the displacement current in the structure is equal to zero, an instantaneous variation of velocity due to an acceleration fluctuation leads immediately to the perturbation of the total current. Therefore, all curves for the total current response start from some non-zero value and then tend to zero at times of about 2 *ps* showing some kind of oscillating behavior (see Fig. 20 (a)). For the circuit operation mode, the total current response induced at the load resistance $R = 10^{-10}\ \Omega m^2$ connected in series with the structure under a constant voltage applied to the whole circuit shows a behavior intermediate between the two limiting cases discussed above (see Fig. 21 (a)). Since the chosen magnitude for the load resistance is about 10 times smaller than the differential resistance of the structure, $Re[Z_d(0)]$, the response characteristics of the total current are quite similar to those of the constant voltage operation (see Fig. 20 (a)). The only difference appears at very short times, since the response in the external circuit has a certain delay time. At time $t = 0$, the fluctuation of the conduction current is entirely compensated by the perturbation of the displacement current, so that $\Delta J = 0$. Therefore, analogously with the case of constant current operation mode, the response function of the total current starts from zero and only after some time delay it follows the response function of the conduction current. The characteristic time of this delay corresponds to the well known RC time. With the increase of R, this delay becomes longer, and the time behavior of the total-current response function becomes more similar to that of the voltage response function reported in Fig. 19 (a).

In accordance with Eq. (81), the Fourier transforms of the response functions considered above give the local values of the generalized IF and AF in the points where the initial perturbations were introduced. The frequency dependence of the square modulus of these generalized IF and AF is to somewhat extent similar for all the three operation modes shown in Figs. 19 (b), 20 (b), 21 (b). Each curve exhibits a plateau in the low frequency

range, followed by a more or less pronounced spike at frequencies over the terahertz region related to plasma oscillations inside the structure, and finally tends to zero at the highest frequency range $f > 5 \div 10\ THz$. As proved in the Appendix, this highest frequency behaviour corresponds to a decrease of the IF and AF squares as ω^{-4} and ω^{-2}, respectively. To illustrate the interrelations between the generalized IF and AF, curves 5 in Figs. 20(b) and 21(b) report the results obtained by recalculating the AF through the IF and the small-signal impedance of the structure according to Eq. (A24) which is the differential analogue of the integral relation given by Eq. (83). The excellent agreement found between the direct and recalculated values is a further test of reliability of the theory so developed.

In accordance with Eq. (80), the spectral density of voltage or current fluctuations can be represented as a spatial integration of the microscopic noise sources with the corresponding generalized IF and AF. The square modulus of the generalized IF for velocity perturbations and the local contribution of velocity accelerations during scattering events into the voltage noise, $|\nabla Z_{\dot{v}}(x_0,\omega)|^2$ and $\delta S_U(x_0,\omega) = n_s(x_0)|\nabla Z_{\dot{v}}(x_0,\omega)|^2 S_{\dot{v}\dot{v}}(x_0)$, are shown in Fig. 21 (a) and (b), respectively, as functions of the coordinate along the structure at frequencies $f =$ 0, 300, 600, 900 GHz (curves 1 to 4). The corresponding spatial profile of the microscopic noise source $S_{\dot{v}\dot{v}}(x_0)$, was already presented in Fig. 13. In full analogy with the conventional IF [37], the generalized IF reaches maximum values at low frequencies in the near-cathode region of the n^+n homojunction. As discussed in Ref. [37] this behavior is due to transit-time effects. At increasing frequencies a cutoff of the transit-time effects takes place and the generalized IF becomes nearly flat inside the n-region at frequency of about 1 THz. Accordingly, the local contribution to the voltage noise comes primarily from the near-cathode n-region. Calculations show also a small additional peak of $\delta S_U(x_0)$ in the near-anode region, which is attributed to the maximum value attained here by the microscopic noise source.

The spatial profiles of the AF square and the local contribution to the conduction current noise calculated under constant voltage operation mode are presented in Fig. 23. The general behavior of these quantities resembles that observed for the corresponding voltage

noise. Moreover, due to the differential relation given by Eq. (A24), the spatial profiles at fixed frequencies are the same in both operation modes. Only the spectral amplitude at fixed x_0 is different, and this difference is determined by the small-signal impedance.

As conclusive step of these calculations, Figs. 24 and 25 report the spectral densities of the fluctuations corresponding to different operation modes. Figure 24 shows the contributions of different microscopic noise sources, connected with velocity and energy accelerations, to the total value of the spectral density of voltage fluctuations. Calculations performed within the generalized IF method are shown together with the results of a direct simulation of voltage noise performed with the MC method. We find that the main contribution to the total spectral density comes from the microscopic noise source related to velocity accelerations during scattering events (curve 2). The cross-correlation between velocity and energy fluctuations during scattering events is found to give a negative contribution to the noise, i.e. it partially compensate the positive contribution belonging to the autocorrelation of energy fluctuations term. Figure 25 presents the spectral density of total current fluctuations for the cases of an unloaded structure and when a load resistance $R = 10^{-10}\ \Omega m^2$ is connected in series with the structure. Of course, in the latter case the total voltage U applied to the whole circuit is chosen in such a way that the stationary voltage drop between the structure terminals is the same as that of the former case. For comparison, symbols show the results obtained by recalculating the corresponding spectral densities with Eq. (83) and $S_J(\omega) = S_U(\omega)/|Z_d(\omega) + R|^2$, and by using the small-signal impedance and spectral density of voltage noise obtained by the same HD approach. The excellent agreement between various HD and MC calculations presented in Figs. 24 and 25 is taken as validation test of the HD-Langevin approach developed here.

To illustrate the advantages of the HD-Langevin scheme, in the following we apply it to a Schottky barrier diode and a p^+n junction as examples of more complicated devices. The results of noise calculations for the $0.35 - 0.35\ \mu m$ GaAs $n^+ - n-$Schottky-contact structure of Ref. [16] are presented in Figs. 26 and 27. Since for Schottky diodes the current noise is of most interest [16], calculations are performed with the AF method.

In so doing, the intermediate quantities such as AF, local contributions, etc. useful for the spatial analysis of the noise are obtained in a natural way. Furthermore, by using the microscopic noise sources originated by scattering events one needs to consider just the noise sources inside the structure, thus avoiding the introduction of bulk and surface noise sources to describe the total noise of the device (see e.g. [54]). Figure 26 compares the spectral densities of current fluctuations calculated by the AF method and the MC procedure at $U = 0.575\ V$ (solid and dashed lines, respectively). As noise source, only the $S_{\dot{v}\dot{v}}-$term is accounted for. This term is sufficient to describe the noise spectrum with high accuracy practically in the whole frequency range. Accordingly, the shot-noise region at low frequencies, the spike in the intermediate frequency range near $f = 600\ GHz$ corresponding to returning carriers [16], as well as the plasma peak in the high-frequency range near $f = 2.2\ THz$ are well reproduced. To outline the advantages given by the generalized transfer fields for the spatial analysis of the noise contribution, Fig. 27 (a) and (b) present the spatial profiles of the AF square and the local contribution to the conduction current noise, $|\nabla Y_{\dot{v}}(x_0,\omega)|^2$ and $\delta S_J(x_0,\omega) = n(x_0)|\nabla Y_{\dot{v}}(x_0,\omega)|^2 S_{\dot{v}\dot{v}}(x_0)$, respectively, calculated as functions of the coordinate along the structure at frequencies $f = 0,\ 500,\ 2500\ GHz$ (curves 1 to 3). In the low-frequency range, the AF reaches a maximum value just near the Schottky barrier, thus indicating that this space region is responsible of shot noise, as expected. In the intermediate frequency range, the non-zero values of the AF cover already the whole n-region. At high frequencies, the AF exhibits a plateau in the n^+ region and some peaks in the n-region reminiscent of the formation of standing waves due to perturbation reflection from the barrier. In summary, the near-barrier region, the n- and n^+-regions are responsible of, respectively, the shot, the returning-carrier and the plasma noises (see Fig. 27 (b) curves 1 to 3).

The results of noise calculations for the bipolar $0.3 - 0.4\ \mu m$ Si p^+n structure of Ref. [14] are presented in Figs. 28 to 30. It should be stressed that some concepts of the conventional IF method become questionable in the presence of bipolar transport. Indeed, even to describe diffusion noise one already needs to introduce two separate IFs, namely: that

for electrons and that for holes. Therefore, the usual interpretation of the IF as a gradient of the small-signal impedance fails. Furthermore, since both electron and hole concentrations enter into Poisson equation, it is no longer straightforward to introduce the electric field as a relevant variable, i.e. to describe its spatio-temporal evolution by a differential equation similar to Eq. (77). In this case it is better to use the concentration, velocity and energy balance equations (i.e. Eqs. (70), (72), (73)) written separately for holes and electrons jointly with the Poisson equation and other equations for the external circuit. The HD-Langevin approach offers a natural way of noise calculations in this case by introducing the separate noise sources in velocity and energy conservation equations (due to the short length of the structure generation-recombination processes are neglected). Again, as noise source, only the $S_{\dot{v}\dot{v}}-$term is accounted for. Figure 28 shows the total spectral density of current fluctuations as well as the hole and electron contributions to the current noise (curves 1 to 3) calculated at a constant applied voltage of $U = 0.7\ V$. Curves correspond to direct calculations by the AF method. Symbols show the result obtained recalculating $S_J(f)$ through Eq. (83) by using $S_U(f)$ and $Z_d(f)$. Analogously to the case of the Schottky-barrier diode considered above, the low-frequency ($f < 1\ GHz$) plateau corresponding to shot-noise is well reproduced for the total as well as for both electron and hole current noise. In the intermediate frequency range ($f = 10 \div 200\ GHz$), a second noise plateau is exhibited by holes while electron current noise show a peaking behavior probably corresponding to a returning carrier mechanism (see also Fig. 29 (a)). At high-frequencies we have found a peak of the hole current noise due to plasma oscillations ($f \approx 1\ THz$). The spatial analysis of the electron and hole current noise (i.e. the local partial contributions to δS_j) at different frequencies $f = 0,\ 100,\ 500,\ 1000\ GHz$ (curves 1 to 4) is illustrated in Figs. 29 (a) and (b), respectively. One can see that, practically at all frequencies, electron noise is determined by the n-region. The spatial map of hole noise is more complicated: shot noise is primarily determined by the holes placed in the n-region; the noise plateau at intermediate frequencies is determined by the holes placed in the n-region just near the p^+n-junction, and only the noise at high-frequencies ($f \geq 500\ GHz$) is determined primarily by the p^+ region.

The low-frequency values of S_J are shown in Fig. 30 as function of the current density j (symbols "HD"). Apart from the value at the highest current, $S_J(0)$ exhibits the $2qj$-behavior (solid line) typical of shot noise for semiconductor devices with exponential current-voltage characteristic under barrier-limited transport. The same behavior is observed for the electron and hole contributions to the total low-frequency current noise plotted in Fig. 30 as functions of the electron and hole currents (symbols "e" and "h", respectively). Thus, the HD-Langevin approach with the microscopic noise sources caused by electron and hole accelerations during scattering events is able to reproduce also shot noise which is usually attributed to the process in which carriers randomly cross a barrier.

E. Concluding remarks

In this section we have developed a hydrodynamic-Langevin approach to calculate electronic noise and associated quantities in deep submicron structures when the length of the active region is of the order or less than the carrier mean free path. In contrast with the conventional IF method (see Sec. 2), where the noise source is represented by velocity fluctuations, here the noise source is represented within an acceleration fluctuation scheme. In this new scheme all the correlations associated with the carrier dynamics between scattering events are removed from the noise source and incorporated only in the transfer fields. As a consequence, we strictly separate the transfer characteristics of the noise due to the spatio-temporal evolution of fluctuations inside the structure from the source of fluctuations. In full analogy with the BTE-Langevin scheme the source of fluctuations is now determined only by the random processes responsible for scattering events. The relevant advantage envisaged by such a representation is that the noise source is now white, i.e. it does not contain any correlations neither in space nor in time. When comparing the present scheme with the conventional approach, we faced the drawback that in the framework of the HD equations such a noise source cannot be described in terms of a single physical quantity. Now, a set of random accelerations is necessary for all the dynamical characteristics of carriers whose

average values enter the system of HD equations. Such accelerations can now contain cross-correlation between themselves, since variations of velocity, energy and higher moments of the distribution function depend on the same scattering event. However, this drawback is proved to be not essential when compared with the advantage of having recovered a white spectrum for the noise source. The spectral power of such a noise source can be treated as depending only on the local mean energy of carriers. Therefore, now all the parameters necessary to close the system of HD equations and to describe the power of noise sources can be obtained from MC simulations (with constant electric field) of the homogeneous bulk materials which compose the structure. Thus, the HD-Langevin approach provides the same closed system of equations to calculate both transport and noise phenomena. The number of HD equations involved in the representation is usually restricted by the level chosen to describe in detail the processes which determine the carrier transport through the structure and, hence, the noise transfer. As an example which illustrates the possibilities of the approach, by taking velocity and energy balance equations here we have presented the calculations of the spatial cross-correlation of the velocity fluctuations under the frozen electric field inside a submicron structure which was used as the noise source in Sec. 2.

A further advantage of the acceleration scheme proposed here is that the noise source is now moved from the field equation (which describes electric field fluctuations in the conventional approach) to the medium equations, i.e. into the HD equations themselves. This allows us to describe all transfer phenomena related to the evolution of fluctuations inside the structure in the framework of a unified formalism. This formalism is based on the construction of functions describing the linear response to a single local perturbation caused by a scattering event, i.e. the Green function formalism. By using the Wiener-Khintchine theorem, the spectral power of fluctuations of arbitrary characteristics, which describe either local properties of the structure or its global behavior, are represented in the same way. This representation is given by an integration over the device volume of a kernel which factorizes three terms related to: (i) the transfer fields determined by the corresponding response functions, (ii) the local carrier concentration, and (iii) the noise source power. The HD

approach has been extended to account for the presence of an external circuit by adding the equations which relate the voltage drop between the structure terminals to the current flowing through the structure. In this way one can directly investigate the noise phenomena when the structure operates under various modes with the external circuits. In particular, the power of the equivalent Norton or Thevenin generators can be described now in the same way by introducing the concepts of generalized impedance and admittance fields for the constant current and constant voltage operations, respectively. In the framework of the acceleration fluctuation scheme, these generalized fields relate the local power of noise sources inside the structure to the power of fluctuations induced by them in the external circuit.

The formal equivalence of the noise description in terms of the Norton and Thevenin generators does not imply a physical equivalence of the evolution of fluctuations inside the structure, which, as shown in the present article, depends on the structure operation mode. Under constant current operation mode the spatio-temporal evolution of fluctuations after their excitation takes a local character covering larger and larger regions of the device due to the drift-diffusion dynamics of carriers. In contrast, under constant voltage operation mode the perturbation expands into the whole volume of the device simultaneously with its initiation by a local source. Such a global evolution of fluctuations, which provides their synhronization in the whole volume of the device at once, is just the reason of formation of some structures such as dipole domains, accumulation layers, etc. under nonlinear evolution of fluctuations.

Let us stress that, even for the linear fluctuations considered in this paper, such a physical difference in the evolution of fluctuations implies that the concepts of transfer field corresponding to different operation modes (such as the IF and the AF), have a independent meaning. Indeed, only under the steady-state conditions in the case of the two-terminal structures the spectral densities of current and voltage fluctuations are interrelated by Eq. (83) through the small-signal impedance. Accordingly, only in this case IF and AF are interrelated by Eq. (A24), see Appendix, i.e. IF can be calculated from the AF and vice

versa. In more general case, when the violation of the symmetry between the voltage and current operation modes takes place, Eqs. (83) and (A24) fail and the concepts of IF and AF play independent roles since they reflect different physical processes and features of the noise transfer from the source to the structure terminals. Therefore, when the current noise operation mode is considered, the concept of an AF proposed in the present work assumes an independent role, being in some cases the most preferable comprehensive deterministic method of noise description from the physical point of view. To some extent this was illustrated by numerical calculations for a Schottky-barrier diode and a p^+n junction taken as typical bipolar device.

IV. FINAL CONCLUSIONS

This paper overviews and implements the transfer-field method as applied for the calculation of electronic noise in deep submicron semiconductor structures. Two basic schemes are used and developed in detail. The former considers the velocity fluctuations and the latter the acceleration fluctuations as microcopic noise sources. We show that the latter scheme has several advantages with respect to the former. Indeed, starting from Markovian noise sources, the latter scheme strictly separates the local noise sources from their time and spatial evolution. In this way, the dual representation of the noise spectral density in terms of impedance and admittance fields is recovered. A remarkable achievement is that from the knowledge of the bulk Langevin sources at a hydrodynamic level it is possible to calculate the noise spectra of non-homogeneous structures even for deep submicron devices. Indeed, non-local effects are associated fully with the transfer fields thus allowing us to calculate the noise sources of a given material in the presence of arbitrary electric field strenghtgs once for ever. This new methodology has been validated by comparison of the present scheme with self-consistent MC approaches for different structures of interest including n^+nn^+ diodes, Schottky diodes, $p-n$ junctions, and different materials like covalent and III-V semiconductor compounds. We believe that the potentiality of the method here developed should be

extended to more complicated structures, as three terminal devices, and eventually include quantum effects.

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APPENDIX A: ANALYTICAL MODEL OF NON-HOMOGENEOUS STRUCTURES

In this appendix the dynamical evolution of perturbations is investigated on the basis of a simplified analitical model for a non-homogeneous structure. In the framework of such a model, some relevant interrelations between various generalized transfer fields, differential impedance, local conductivity, etc. are obtained.

1. Evolution of fluctuations

Let us consider an unipolar structure with a non-homogeneous doping profile. To simplify the problem we further assume that any fluctuation of the carrier energy is absent, thus we omit the energy conservation equation (Eq. (73)). In this case the three phenomenological parameters entering the velocity conservation equation (Eq. (72)), namely, the average reciprocal effective mass m^{-1}, the velocity relaxation rate ν_v and the variance of velocity fluctuations Q_v responsible for the spatial diffusion, will depend on the spatial coordinate through the steady-state spatial profile of the mean energy $\varepsilon_s(x)$. In the framework of such a model, the system of equations describing the linear characteristics of the structure can be obtained by linearizing Eqs. (72), (77), and (78). As a result, for the fluctuations of, respectively, velocity Δv, electric field ΔE and carrier concentration Δn, one obtains:

$$\frac{d\Delta v}{dt} + (\nu_v + \frac{\partial v_s}{\partial x})\Delta v + Q_v \frac{\partial}{\partial x}\frac{\Delta n}{n_s} = e m^{-1} \Delta E + \frac{\xi_v}{n_s} \qquad (A1)$$

$$\frac{d\Delta E}{dt} + \frac{e n_s}{\epsilon\epsilon_0}\Delta v = \frac{\Delta J}{\epsilon\epsilon_0} \qquad (A2)$$

$$\Delta n = \frac{e}{\epsilon\epsilon_0}\frac{\partial}{\partial x}\Delta E \qquad (A3)$$

where ΔJ is the fluctuation of the total current in the external circuit, $\frac{d}{dt} = \frac{\partial}{\partial t} + v_s \frac{\partial}{\partial x}$ is the full time derivative of some quantity in the neighborhood of point x.

By taking the $\frac{d}{dt}$ of both sides of Eq. (A2) and then eliminating $\frac{d\Delta v}{dt}$, Δv and Δn with the help of Eqs. (A1), (A2) and (A3), one finally obtains the equation which describes

the spatio-temporal evolution of the electric field fluctuations, $\Delta E(x,t)$, induced by the primitive noise source ξ_v under the condition that the structure is arbitrarily connected with an external circuit. In operator form this equation can be written as:

$$\hat{A}\{\Delta E(x,t)\} = \frac{1}{\epsilon\epsilon_0}\left[\frac{\partial}{\partial t}\Delta J + \tilde{\nu}(x)\Delta J - e\xi_v\right] \tag{A4}$$

where

$$\hat{A}\{\ \ \} = \frac{d^2}{dt^2} - Q_v n_s \frac{\partial}{\partial x}\left[\frac{1}{n_s}\frac{\partial}{\partial x}\right] + \tilde{\nu}(x)\frac{d}{dt} + \omega_p^2(x) \tag{A5}$$

is the linear operator of the evolution, and $\omega_p^2(x) = e^2 n_s/\epsilon\epsilon_0 m$ and $\tilde{\nu}(x) = \nu_v + n_s \frac{\partial}{\partial x}\frac{v_s}{n_s}$ are the local values of the plasma frequency and the effective relaxation rate of the velocity in the neighborhood of point x, respectively.

To write the formal solution of the equation $\hat{A}\{y\} = f(x,t)$, where f is an arbitrary function, we shall use the method of the inverse operator by introducing $\hat{A}^{-1}$ which gives the solution in the form: $y = \hat{A}^{-1}\{f\}$. In the time domain representation (stressed by subindex t) it is:

$$\hat{A}_t^{-1}\{f(t)\} = \int_0^L dx_0 \int_0^\infty ds G(x, x_0, s) f(x_0, t-s) \tag{A6}$$

where $G(x, x_0, s)$ is the Green function of the evolution operator of Eq. (A6), i.e. $\hat{A}_t^{-1} \equiv G(x, x_0, s)$. In the frequency domain representation (stressed by subindex ω) Eq. (A6) can be written as:

$$\hat{A}_\omega^{-1}\{f(\omega)\} = \int_0^L dx_0 G(x, x_0, \omega) f(x_0, \omega) \tag{A7}$$

where $\hat{A}_\omega^{-1} \equiv G(x, x_0, \omega) = \int_0^\infty G(x, x_0, s) exp(-i\omega s) ds$ and $f(x,\omega) = \int_{-\infty}^{\infty} f(x,t) exp(-i\omega t) dt$.

Let us outline some properties of the evolution operator, which will be used later. When $f(x,\omega)$ is independent of x, from Eq. (A5) it follows that

$$\hat{A}_\omega\{f(\omega)\} = [-\omega^2 + i\omega\tilde{\nu}(x) + \omega_p^2(x)] f(\omega) \tag{A8}$$

By applying $\hat{A}_\omega^{-1}$ to both sides of Eq. (A8) and taking $f(\omega) \equiv 1$ one obtains the equality

$$\hat{A}_\omega^{-1}\{-\omega^2 + i\omega\tilde{\nu}(x_0) + \omega_p^2(x_0)\} = 1 \tag{A9}$$

For the case of a homogeneously doped structure, when $\tilde{\nu}$ and ω_p are independent of x, from Eq. (A9) one obtains:

$$\hat{A}_\omega^{-1}\{\ \ \} = 1/L(-\omega^2 + i\omega\tilde{\nu}_v + \omega_p^2) \qquad (A10)$$

Within this notation, the formal solution of Eq. (A4) will contain two terms, thus indicating the presence of two essentially different mechanisms which govern the spatio-temporal evolution of the electric field in the structure connected with an external circuit:

$$\Delta E(x,\omega) = \Delta E_\xi(x,\omega) + \Delta E_J(x,\omega) \qquad (A11)$$

Here the first term

$$\Delta E_\xi(x,\omega) = -\frac{e}{\epsilon\epsilon_0}\hat{A}_\omega^{-1}\{\xi_v(x_0,\omega)\} \qquad (A12)$$

is the part of the electric field fluctuation which from the point of view of Eq. (A4) is originated by the primitive noise source ξ_v, i.e. by a scattering event. This contribution does not dependent on the external circuit.

The second term

$$\Delta E_J(x,\omega) = \frac{1}{\epsilon\epsilon_0}\hat{A}_\omega^{-1}\{i\omega + \tilde{\nu}(x_0)\}\Delta J(\omega) \qquad (A13)$$

describes the field fluctuations which are determined by the external circuit and have no direct connection with the noise source ξ_v. Formally, in Eq. (A13) the source of fluctuations is represented by the current perturbation in the external circuit, $\Delta J(\omega)$, which the circuit allows to pass through itself.

Under constant current operation mode, when $\Delta J = 0$, a perturbation appearing in the structure does not pass to the external circuit as some fluctuation of ΔJ. Therefore, any influence of the external circuit on the evolution of a fluctuation inside the structure is absent, $\Delta E_J(x,\omega) = 0$. In this case the evolution of a fluctuation after its appearance in some point x_0 of the structure will be described by the Green function $\hat{A}_\omega^{-1}$ of the evolution operator $\hat{A}\{\ \}$ given by Eq. (A5). When $\Delta J = 0$ the fluctuation evolution occurs under the conditions of complete local compensation of conduction with displacement current perturbations, $\Delta j_{cond}(x,t) + \Delta j_{disp}(x,t) = 0$, respectively given by, $\Delta j_{cond}(x,t) = e(n_s\Delta v + \Delta n v_s)$

and $\Delta j_{disp}(x,t) = \epsilon\epsilon_0 \frac{\partial}{\partial t}\Delta E$. This means that the power associated with these fluctuations, $P(x,\omega) = \Delta j^*_{cond}(x,\omega)\Delta E(x,\omega) = i\omega|\Delta E(x,\omega)|^2\epsilon\epsilon_0$, will contain the reactive part only. As a result of such a compensation, the propagation of the perturbation inside the structure occurs through structure regions which are locally coupled and gradually cover more and more volume around the point where the perturbation was initiated. This can be seen from Eq. (A5) for the evolution operator. For example, in the homogeneous case, when n_s and v_s are independent of x, after the initialization of the perturbation in some point x_0, plasma oscillations with frequency ω_p are induced in this point. During the initial stage of the perturbation evolution, a fast growth of a narrow peak of ΔE near the point x_0 will take place. Due to diffusion broadening the left and right wings of the peak try to move in opposite directions with velocities $v_s - \sqrt{Q_v}$ and $v_s + \sqrt{Q_v}$, respectively. As a result, the initial peak is split into two independent parts which move in opposite directions. Inside the region placed between the two peaks, moving away plasma oscillations create a new peak of ΔE which after a while will be split again by diffusion into two peaks moving in opposite directions. Such a process will be continued until the relaxation of velocity will damp the plasma oscillations.

Let us consider now the situation when $\Delta J \neq 0$, i.e. the fluctuation appearing inside the structure moves outside into the external circuit. As follows from Eq. (A13), the source of fluctuations of the field component $\Delta E_J(x,\omega)$ is represented by the macroscopic fluctuations of the total current in the circuit, ΔJ, in place of the microscopic noise source inside the structure, which really induces the fluctuation ΔJ. Such a seeming replacement of the noise source is caused by the fact that when the external circuit allows for the fluctuation to move outside the structure, the local constraint of total current conservation $\Delta J = j_{disp}(x,t) + j_{cond}(x,t)$ leads to a synchronization of the time variations of the electric field $\Delta E(x,t)$ and of the conduction current $j_{cond}(x,t)$ in each point of the structure. The field component $\Delta E_\xi(x,\omega)$ alone cannot provide such a synchronization due to the local character of its spatial evolution. As a consequence, when $\Delta J \neq 0$, there must appear an additional

component of the electric field, $\Delta E_J(x,\omega)$, driven by a current fluctuation in the external circuit, ΔJ. In turn, this is the component which provides the synchronization required by total current conservation for the simultaneous process of a perturbation evolution on the whole structure just after the initialization of the fluctuation by some primitive noise source. Thus, the appearance of the additional component of the electric field perturbation, $\Delta E_J(x,\omega)$, during the process of the single-fluctuation evolution can be considered as the manifestation of a feedback process through the external circuit. On a macro-level, this feedback transforms the spatially local character of the fluctuation evolution into a global one by expanding the perturbation into the whole structure at once.

Let us show that, in the general case, the spatial profile of $\Delta E_J(x,\omega)$ is determined by the spatial profile of the differential impedance of the structure, $\nabla Z'(x,\omega)$. For this sake we consider the case when the structure is connected with an external circuit characterized by the input complex impedance $Z(\omega)$. In the frequency representation the linearized equation of such a circuit takes the form:

$$\Delta J(\omega)Z(\omega) = -\Delta U_d(\omega) \equiv -\int_0^L \Delta E(x,\omega)dx \qquad (A14)$$

By substituting into the r.h.s. of Eq. (A14) $\Delta E(x,\omega)$ given by Eq. (A11) and taking into account the spatial dependence of the field component $\Delta E_J(x,\omega)$ given by Eq. (A13) one obtains the well-known expression for the current fluctuations in the circuit characterized by the impedance $Z(\omega)$:

$$\Delta J(\omega) = \frac{\Delta U_N(\omega)}{Z_d(\omega) + Z(\omega)} \qquad (A15)$$

written in terms of voltage fluctuations given by the Norton generator

$$\Delta U_N(\omega) = \left[\frac{e}{\epsilon\epsilon_0}\int_0^L dx \hat{A}_\omega^{-1}\right]\{\xi_v(x_0,\omega)\} \qquad (A16)$$

and the small-signal impedance of the structure

$$Z_d(\omega) = \frac{\int_0^L \Delta E_J(x,\omega)dx}{\Delta J(\omega)} = \int_0^L \nabla Z'(x,\omega)dx \qquad (A17)$$

where

$$\nabla Z'(x,\omega) = \frac{1}{\epsilon\epsilon_0}\hat{A}_\omega^{-1}\{i\omega + \tilde{\nu}(x_0)\} \qquad (A18)$$

is the differential impedance of the structure. Thus, by comparing Eqs. (A13) and (A18) one can conclude that $\Delta E_J(x,\omega)$ is the distribution of the potential drops on the paths dx, characterized by the local differential impedance $\nabla Z'(x,\omega)dx$, which are caused by ΔJ.

By substituting Eq. (A15) into Eq. (A13) one obtains the final expression which describes the macroscopic fluctuations of the electric field in the structure initiated by the primitive noise source ξ_v:

$$\Delta E(x,\omega) = -\frac{e}{\epsilon\epsilon_0}\left[\hat{A}_\omega^{-1} - \frac{\nabla Z'(x,\omega)}{Z_d(\omega)+Z(\omega)}\int_0^L dx\hat{A}_\omega^{-1}\right]\{\xi_v(x_0,\omega)\} \qquad (A19)$$

where the noise source $\{\xi_v(x_0,\omega)\}$ which is subjected to the action of the inverse evolution operator $\hat{A}_\omega^{-1}$ is on purpose placed out of the square brackets to stress that the expession inside the square brackets is the Green function describing the response of the electric field ΔE when the initial perturbation has occurred at point x_0.

By using the constraint of total current conservation, similarly to Eq. (A19) one obtains the expression for the fluctuations of the conduction current component inside the structure:

$$\Delta j_{cond}(x,\omega) = \Delta J - i\omega\epsilon\epsilon_0\Delta E(x,\omega) =$$
$$e\left[i\omega\hat{A}_\omega^{-1} + \frac{\sigma(x,\omega)}{\epsilon\epsilon_0}\frac{\nabla Z'(x,\omega)}{Z_d(\omega)+Z(\omega)}\int_0^L dx\hat{A}_\omega^{-1}\right]\{\xi_v(x_0,\omega)\} \qquad (A20)$$

where

$$\sigma(x,\omega) = \frac{1}{\nabla Z'(x,\omega)} - i\omega\epsilon\epsilon_0 \qquad (A21)$$

has, as we shall see below, the physical meaning of local conductivity of the structure.

2. Transfer fields

Equations (A19) and (A20) allows us to obtain the expressions of the generalized transfer fields for both the constant current ($Z(\omega)=\infty$) and constant voltage ($Z(\omega)=0$) operation modes. As follows from Eq. (A19), under constant current operation mode the spectral

representation of $\hat{A}_\omega^{-1}$ determines the generalized IF which enters Eq. (80). In terms of the present notation, the IF can be written as:

$$\nabla Z_{\dot{v}}(x_0,\omega) = \frac{e}{\epsilon\epsilon_0}\int_0^L dx\hat{A}_\omega^{-1} \equiv \frac{e}{\epsilon\epsilon_0}\int_0^L dx G(x,x_0,\omega) \qquad (A22)$$

For the constant voltage operation mode one obtains from Eq. (A20) an expression for the AF in the form:

$$\nabla Y_{\dot{v}}(x_0,\omega) = \frac{1}{L}\left[i\omega\epsilon\epsilon_0 + \frac{1}{Z_d(\omega)}\int_0^L \sigma(x,\omega)\nabla Z'(x,\omega)dx\right]\frac{e}{\epsilon\epsilon_0}\int_0^L dx\hat{A}_\omega^{-1} \qquad (A23)$$

By using Eq. (A21) it is easy to show that the first multiplier in Eq. (A23) corresponds to $Z_d^{-1}(\omega)$. As a result, one obtains the relation which connects the IF and AF for the same primitive perturbation applied in the point x_0 as:

$$\nabla Y_{\dot{v}}(x_0,\omega) = \frac{\nabla Z_{\dot{v}}(x_0,\omega)}{Z_d(\omega)} \qquad (A24)$$

Eq. (A24) can be considered as the differential analogue of Eq. (83), written for the single local perturbation rather than for the spectral power of the Norton or Thevenin noise generators already integrated over the whole structure.

3. The local model

By replacing in the second term of Eq. (A20) $\Delta J(\omega)$ given by Eq. (A15) by the same factor extracted from Eq. (A19) (the second term in the square brackets), one obtains the local relation between the fluctuations of the conduction current and those of the electric field in a certain point of the structure:

$$\Delta j_{cond}(x,\omega) = \Delta\tilde{j}_{cond}(x,\omega) + \sigma(x,\omega)\Delta E(x,\omega) \qquad (A25)$$

where

$$\Delta\tilde{j}_{cond}(x,\omega) = \frac{e}{\epsilon\epsilon_0}\frac{\hat{A}_\omega^{-1}\{\xi_v(x_0,\omega)\}}{\nabla Z'(x,\omega)} \equiv -\frac{\Delta E_\xi(x,\omega)}{\nabla Z'(x,\omega)} \qquad (A26)$$

is the local conduction current component directly initiated by the primitive source and $\sigma(x,\omega)$ is the local conductivity of the structure given by Eq. (A21) describing the part of

the conduction current induced by the electric field fluctuations appeared in the structure around the point x.

The relation given by Eq. (A21) allows us to give the following interpretation for the equivalent scheme of a non-homogeneous structure. Let us represent the whole structure as a sequence of N resistors connected in series, each of length $\Delta x = L/N$ and internal complex resistance $\Delta R(x_m, \omega) = \Delta x/[S\sigma(x_m, \omega)]$, and in parallel with each resistor a capacitance $\Delta C = \epsilon\epsilon_0 S/\Delta x$. Here $x_m = (m - 0.5)\Delta x$, $m = 1,\ 2 ..., N$; S is the structure cross-section, which in this article is taken to be equal to unity. The total impedance of such a circuit is:

$$Z(\omega) = \lim_{N\to\infty} \sum_{m=1}^{N} [\Delta R^{-1}(x_m, \omega) + i\omega\Delta C]^{-1} \equiv Z_d(\omega) \qquad (A27)$$

which in the limit $N \to \infty$ transforms into the definition of the small-signal impedance of the structure given by Eq. (A17).

Within the above equivalent circuit, $\Delta E_\xi(x, \omega)$ given by Eq. (A12) and $\Delta\tilde{j}_{cond}(x, \omega)$ given by Eq. (A26) will play the role of two alternative representations of the local noise sources corresponding to the Norton and Thenevin generators. Analogously with the case of the equivalent generators which describe the noise of the whole structure, in the framework of the local representation the spectral powers of these generators are related through an expression similar to Eq. (83) where now, instead of the impedance of the whole structure, we have the differential impedance in the neighborhood of point x.

4. Influence of energy fluctuations

The above analytical model was developed by neglecting energy fluctuations. The inclusion of energy fluctuations and/or higher-order moments of the distribution function does not change the main conclusions and the relations obtained within the simple model. Indeed, such an inclusion will result in adding to Eq. (A4) an equation describing energy fluctuations with its own independent noise source. As a consequence we shall have a coupled system of equations. Each equation will contain additional cross-terms which describe the mutual influence of the fluctuations of the electric field and energy. In the framework of such a system,

the response function of the electric field, $G_\alpha(x, x_0, s)$, will contain already two components with $\alpha = \dot{v}$ and $\dot{\varepsilon}$, describing the response to the instantaneous variations of the velocity and energy. As a consequence, in the expressions which describe the fluctuations of ΔE and Δj_{cond}, namely in Eqs. (A12), (A16), (A19) and (A20), there will appear a summation over the different sources, i.e. $\hat{A}_\omega^{-1}$ will be replaced by $\sum_{\alpha=\dot{v},\dot{\varepsilon}} \hat{A}_{\omega,\alpha}^{-1}$. Analogously to the case when energy fluctuations are neglected, the definition of the differential impedance (see Eq. (A18)), and hence the small-signal impedance of the structure, will contain only the component of the response functions due to velocity perturbations. As a result, in the framework of the acceleration fluctuation scheme the differential relation between the generalized IF and AF given by Eq. (A24) will remain valid for any source of perturbation.

5. Comments on the conventional IF method

The equivalent circuit scheme of a non-homogeneous structure considered in Sec. 3 of this Appendix and the possibility to represent, in the framework of this scheme, the local noise sources in terms of current and voltage generators allow us to put some remarks concerning the internal inconsistency of the conventional IF method, when the noise source is represented as the current fluctuations appearing in the structure. For this sake, let us firstly use Eqs. (A25) and (A26) to determine the fluctuations of the voltage drop between the structure terminals, $\Delta U_d(\omega)$. Jointly with the total current conservation equation $i\omega\epsilon\epsilon_0\Delta E(x,\omega)+\Delta j_{cond}(x,\omega) = \Delta J$, these equations allow us to write at once the expression for $\Delta U_d(\omega)$ in terms of the local conductivity $\sigma(x,\omega)$ or, by using additionally Eq. (A21), through the differential impedance $\nabla Z'(x,\omega)$, as:

$$\Delta U_d(\omega) = \int_0^L \frac{\Delta\tilde{j}_{cond}(x,\omega)}{i\omega\epsilon\epsilon_0 + \sigma(x,\omega)}dx = \int_0^L \nabla Z'(x,\omega)\Delta\tilde{j}_{cond}(x,\omega)dx \qquad (A28)$$

By applying the Wiener-Khintchine theorem to Eq. (A28) and making the usual assumption that the noise sources are uncorrelated in space (this assumption helps to simplify but it is

not strictly necessary) one obtains the Shockley IF formula

$$S_U(\omega) = \int_0^L |\nabla Z(x,\omega)|^2 S_{\tilde{j}}(x,\omega)dx \qquad (A29)$$

In this case, $S_{\tilde{j}}(x,\omega)$ is the spectral power of the local current generator, represented by the stochastic component of the conduction current, $\Delta\tilde{j}_{cond}(x,\omega)$, and the role of the IF is performed by the differential impedance $\nabla Z(x,\omega) = \nabla Z'(x,\omega)$. The representation of $S_U(\omega)$ by Eq. (A29) meets with an evident inconsistency when in the framework of the same treatment of the noise source as local generator of current one has at least two representations for the IF which relates the local noise source with the fluctuations of the voltage drop between the structure terminals. A commonly adopted concept of the IF as basically formulated in [58, 62, 64], is to define $\nabla Z(x,\omega)$ and $\nabla Z'(x,\omega)$ as different characteristics of the structure, even if they are interconnected by the simple relations: $Z_d(\omega) = -\int_0^L \nabla Z(x,\omega)dx = \int_0^L \nabla Z'(x,\omega)dx$. The essence of this inconsistency does not imply that one of the approaches used to determine the IF in Eq. (A29) is wrong. The essence is that the source of fluctuations, i.e. $S_{\tilde{j}}(x,\omega)$, in each of the approaches is different even if the same notion, i.e. current fluctuations, is used for its characterization. In the case considered here, the source of fluctuation is the stochastic part of the conduction current fluctuations, i.e. $\Delta\tilde{j}_{cond}(x,\omega)$ in Eq. (A25). It is easy to see that such a source cannot independently initiate the transfer of an electric field fluctuation through the structure, since this source by itself is already a result of such an evolution (see Eq. (A26)). Indeed, formally, the equation which describes the evolution of ΔE-fluctuations for such a source coincides with the total current conservation law in the structure, which describes only the local interrelation between ΔE and $\Delta\tilde{j}_{cond}$, i.e. it does not contain terms responsible for the spatial evolution of ΔE-perturbations in the structure volume. As a result, a further evolution of ΔE-perturbation after its initiation by the noise source will take the form of locally damped plasma oscillations in the initiation point and the contribution of the source into $\Delta U_d(\omega)$ will be determined by the differential impedance of the structure taken at the perturbation point.

As it was shown in Sec. 2, the conventional IF in the commonly adopted treatment [58, 62, 64] can be comprehensively introduced starting from the microscopic level of description, when the noise source is represented by the conduction current fluctuations associated with velocity fluctuations only. The remaining part of the conduction current fluctuations associated with fluctuations of carrier concentration Δn, must be included into the field equation which describes the ΔE-perturbation evolution (for example, in Eqs. (15) or (A2) it is the convective term $v_s \partial \Delta E / \partial x$). In this case, the diffusion broadening and the drift of any fluctuation of concentration provides the propagation inside the structure volume of ΔE-perturbations initiated by the local source. As a result, to describe the transfer of fluctuations up to the structure terminals one must use the IF rather than the differential impedance.

¿From the analysis presented here of the two cases in which $S_U(\omega)$ can be represented by Eq. (A29), one can see that in the first and second case we deal with sources which are different in a physical sense, even if, due to historical reasons, they are characterized in terms of the same notion of *conduction current fluctuations*. It is evident that, for different sources, their functional relation with fluctuations of the voltage drop between the structure terminals can be different keeping at the same time an adequate description of $S_U(\omega)$. The examples considered above demonstrate the physical ambiguity of the conventional IF method in the definition of the basic notions (i.e. the noise source and, hence, the IF), an ambiguity leading to internal contradictions inherent to this method. ¿From one side, the method implies a separate description of both the noise source and the processes of further transfer of the noise to the structure terminals. ¿From another side, the concept of the method is based on the replacement of the actual noise source, determined by carrier scattering events, with certain other sources which already include also part of processes responsible for the further evolution of fluctuations after their initiation. Thus, the central idea of separating the source of noise from its transfer becomes very confused and the individuation of the correct noise sources is a rather delicate task. Therefore, the correct application of the IF method requires a rather strict consistency of the approaches used for the determination

of both the IF in itself and the noise source, as it was illustrated in Sec. 2 for the case of micron and submicron structures.

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FIGURE CAPTIONS

Fig. 1 - Cross-correlation functions $C_{\delta\tilde{j}}(x' - x'', s)$ calculated by Eq. (45) as function of the dimensionless time γs for normalized distances $(x' - x'')\gamma^2 m/eE = 0.05,\ -0.05,\ 0.1,\ -0.1,\ 0.2,\ -0.2$ (respectively curves 1 to 6).

Fig. 2 - Spatial profiles of: (a) concentration, (b) average velocity, (c) mean energy and (d) electric field calculated by HD (solid lines) and MC (dots) approaches for a $0.21 - 0.3 - 0.39$ μm n^+nn^+ GaAs structure with dopings $n = 5 \times 10^{15}$ and $n^+ = 10^{17}$ cm^{-3}. $T = 300$ K. The average voltage drop between the structure terminas is $U_d = 0.5$ V.

Fig. 3 - (a) Average velocity and (b) mean energy as functions of the local electric field calculated by the MC method in bulk GaAs (curve 1), by the HD approach in a long GaAs structure (curve 2), and by the HD and MC procedures in the submicron structure of Fig. 2 (curves 3 and 4, respectively). The long structure corresponds to a $0.5 - 7.5 - 0.5$ μm n^+nn^+ GaAs structure with doping levels $n = 2 \times 10^{14}$ cm^{-3} and $n^+ = 10^{16}$ cm^{-3} at $T = 300$ K and $U_d = 8.0$ V.

Fig. 4 - Spatial profiles of the real (curves 1, 2) and imaginary (curves 3, 4) parts of $\nabla Z(x_0, f)$ at frequencies of 25 (curves 1, 3) and 225 GHz (curves 2, 4) calculated by the HD approach for the submicron structure of Fig. 2.

Fig. 5 - Spatial profiles of $|\nabla Z(x_0, f)|^2$ calculated at frequencies $f = 0,\ 5,\ 10,\ 15,\ 20,\ 25$ GHz (curves 1 to 6, respectively) by the HD approach for the long structure of Fig. 3.

Fig. 6 - (a) Auto- and (b) cross-correlation functions of conduction current fluctuations, $C_{\delta\tilde{j}}(n, n, s)$ and $C_{\delta\tilde{j}}(n, n+2, s)$, respectively, calculated by the MC method for the short structure of Fig. 2. Only values corresponding to positive times are shown. Curves 1 to 4 correspond respectively to the cell numbers $n = 10,\ 12,\ 14,\ 16$.

Fig. 7 - Cross-correlation functions of conduction current fluctuations including only velocity fluctuations, $C_{\delta\tilde{j}}(n,m,s)$ calculated by the MC method for the short structure of Fig. 2. Only values corresponding to positive times are shown. Curves 1 to 4 correspond to $n = 13$ and, respectively, to $m = 9,\ 11,\ 15,\ 17$.

Fig. 8 - Cross-correlation functions of conduction current fluctuations: (a) including only velocity fluctuations $C_{\delta\tilde{j}}(n,m,s)$ and (b) including both velocity and number fluctuations $C_{\delta j}(n,m,s)$ calculated by the MC method for the short structure of Fig. 2. Curves 1 to 5 correspond to $n = 10$ and, respectively, to $m = 10,\ 12,\ 14,\ 16,\ 18$.

Fig. 9 - Spectral behavior of the real (curves 1, 2) and imaginary (curves 3, 4) parts of the noise source $K(n,m,f)$ for the pairs of cells $(n-2,n)$ (curves 1, 3) and $(n,n+2)$ (curves 2, 4), with $n = 14$. The noise source corresponds to conduction-current fluctuations associated only with velocity fluctuations.

Fig. 10 - Spectral density of voltage fluctuations for the long GaAs structure of Fig. 3 calculated by the MC method (curve 1), as well as by the conventional IF approach based on the HD model with (curve 2) and without (curve 3) the frequency dependence of the velocity fluctuations noise source $S_v(\omega)$ evaluated from the same HD model.

Fig. 11 - Frequency dependence of the spectral density of voltage fluctuations of the submicron structure of Fig. 2 calculated by using different techniques: MC simulation (curve 1), IF method neglecting spatial correlations (curve 2), IF method including spatial correlations and considering a noise source related only to velocity fluctuations (curve 3) as well as to number and velocity fluctuations (curve 4).

Fig. 12 - Energy dependence of the spectral density of the microscopic noise sources $S_{\dot{v}\dot{v}}$, $S_{\dot{v}\dot{\varepsilon}}$, and $S_{\dot{\varepsilon}\dot{\varepsilon}}$ (respectively curves 1 to 3) calculated by the MC simulation of homogeneous bulk GaAs. We remark the frequency independence of the above spectra.

Fig. 13 - Spatial profiles of microscopic noise sources $S_{\dot{v}\dot{v}}$, $S_{\dot{v}\dot{\varepsilon}}$, and $S_{\dot{\varepsilon}\dot{\varepsilon}}$ (respectively curves 1 to 3) calculated for the same $0.21 - 0.30 - 0.39$ μm n^+nn^+ GaAs structure of Sec. 2 of this series with the same parameters: doping levels $n = 5 \times 10^{15}$ cm^{-3} and $n^+ = 10^{17}$ cm^{-3} at $T = 300$ K for a voltage of 0.5 V.

Fig. 14 - Spatial profiles of (a) velocity, (b) concentration, (c) conduction current perturbations and (d) perturbation of conduction current component corresponding to velocity fluctuations only, calculated under the frozen electric field conditions at different time moments when the initial perturbation of velocity is placed at the point $x_0 = 0.315$ μm of the same submicron GaAs structure of Fig. 13.

Fig. 15 - Cross-correlation functions of conduction current fluctuations associated with velocity fluctuations only $C(n, m, t)$ calculated under static field conditions by MC (curves 1 and 2) and HD (curves 3 and 4) methods for cells m centered at the points $x = 0.435,\ 0.495$ μm (curves 1,3 and 2,4, respectively) and a cell n centered at $x = 0.315$ μm in the same submicron structure of Fig. 13.

Fig. 16 - Frequency dependence of the spectral density of voltage fluctuations of the submicron structure calculated by using direct MC simulations (curve 1) and IF method including spatial correlations obtained from the present HD modeling and considering all the noise sources (curve 2). To evaluate the role of the different noise sources curve 3 shows the contribution coming from instantaneous velocity accelerations only determined by the $S_{\dot{v}\dot{v}}$-term in Eq. (77).

Fig. 17 - Spatio-temporal evolution of (a) electric field and (b) conduction current perturbations induced by the initial perturbation of velocity placed at the point $x_0 = 0.315$ μm of the same submicron structure of Fig. 13 under constant total current flowing through the structure.

Fig. 18 - Spatio-temporal evolution of (a) electric field and (b) conduction current perturba-

tions induced by the initial perturbation of velocity placed at the point $x_0 = 0.315\ \mu m$ of the same submicron structure of Fig. 13 under constant voltage applied between the structure terminals.

Fig. 19 - (a) Generalized voltage response functions calculated under constant current operation mode and (b) square modulus of corresponding generalized impedance field. Curves 1 to 4 correspond to initial perturbations of velocity placed, respectively, in points $x_0 = 0.105,\ 0.225,\ 0.315,\ 0.435\ \mu m$ of the same submicron structure of Fig. 13.

Fig. 20 - (a) Generalized total-current response functions calculated under constant voltage operation mode and (b) square modulus of corresponding generalized admittance field. Curves 1 to 4 correspond to initial perturbations of velocity placed, respectively, in points $x_0 = 0.105,\ 0.225,\ 0.315,\ 0.435\ \mu m$ of the same submicron structure of Fig. 13. Curve 5 gives the AF recalculated from Eq. (A24) by using the small-signal impedance of the structure and the generalized IF of Fig. 19 (b).

Fig. 21 - (a) Generalized total-current response functions induced at the load resistance $R = 10^{-10}\ \Omega m^2$ connected in series with the structure and (b) square modulus of corresponding generalized admittance field. Calculations are performed by the present HD approach under a constant voltage applied to the whole circuit. Curves 1 to 4 correspond to initial perturbations of velocity placed, respectively, in points $x_0 = 0.105,\ 0.225,\ 0.315,\ 0.435\ \mu m$ of the same submicron structure of Fig. 13. Curve 5 gives the AF recalculated from equation: $\nabla Y_{\dot{v}}(x_0, \omega) = \nabla Z_{\dot{v}}(x_0, \omega)/[Z_d(\omega) + R]$

Fig. 22 - Spatial profiles of: (a) square modulus of generalized impedance field for velocity perturbations, and (b) local contribution of velocity accelerations during scattering events into the voltage noise, calculated by the present HD-Langevin approach under constant current operation mode. Curves 1 to 4 correspond, respectively, to frequencies $f = 0,\ 300,\ 600,\ 900\ GHz$

Fig. 23 - Spatial profiles of: (a) square modulus of generalized admittance field for velocity perturbations, and (b) local contribution of velocity accelerations during scattering events into the current noise, calculated by the present HD-Langevin approach under constant voltage operation mode. Curves 1 to 4 correspond, respectively, to frequencies $f = 0,\ 300,\ 600,\ 900\ GHz$

Fig. 24 - Spectral density of voltage fluctuations calculated by the present generalized IF method for the structure of Fig. 13 (curve 1) and contributions coming from velocity-velocity, velocity-energy and energy-energy microscopic noise sources (curves 2 to 4, respectively). Curve 5 shows the result of direct simulations of voltage noise by the MC method.

Fig. 25 - Spectral density of current fluctuations calculated directly by the present generalized AF method for the unloaded structure of Fig. 13 (curve 1) and with the load resistance $R = 10^{-10}\ \Omega m^2$ connected in series with the structure (curve 2). Curves 3 and 4 show the results of recalculations of corresponding spectral densities by using the small-signal impedance and spectral density of voltage noise obtained by the same HD approach.

Fig. 26 - Spectral density of current fluctuations calculated for $0.35 - 0.35\ \mu m$ GaAs $n^+ - n-$Schottky-contact structure by the HD-Langevin and MC approaches. $n^+ = 10^{17}$, $n = 10^{16}\ cm^{-3}$. $U_d = 0.575\ V$.

Fig. 27 - Spatial profiles of: (a) square modulus of generalized admittance field for velocity perturbations, and (b) local contribution of velocity accelerations during scattering events into the current noise. Calculations refer to the present HD-Langevin approach under constant voltage operation mode for the Schottky-barrier structure of Fig. 26. Curves 1 to 3 correspond, respectively, to frequencies $f = 0,\ 500,\ 2500\ GHz$

Fig. 28 - Spectral density of total, hole and electron current fluctuations (curves 1 to 3, respectively) calculated by the AF (curves) and IF (symbols) methods for bipolar $0.3-0.4\ \mu m$ Si p^+n diode with $p^+ = 10^{17}$, $n = 5 \times 10^{15}\ cm^{-3}$. $U_d = 0.7\ V$, $j = 6.4 \times 10^6\ A/m^2$.

Fig. 29 - Spatial profiles of local contribution of: (a) electron and (b) hole velocity accelerations during scattering events into the current noise. Calculations refer to the present HD-Langevin approach under constant voltage operation mode for the bipolar structure of Fig. 28. Curves 1 to 4 correspond, respectively, to frequencies $f = 0,\ 100,\ 500,\ 1000\ GHz$.

Fig. 30 - Low-frequency value of the spectral density of current fluctuations as a function of the total current flowing through the bipolar structure of Fig. 28 (symbols "HD") together with the shot-noise expression $S_j = 2qj$ (solid line). For comparison, the curves labelled as "e" and "h" show the $S_J(0)$ of electrons and holes plotted as functions of electron and hole currents, respectively.

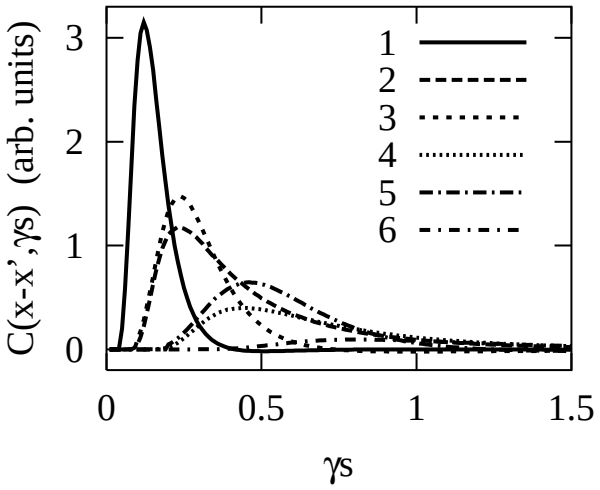

FIG. 1.

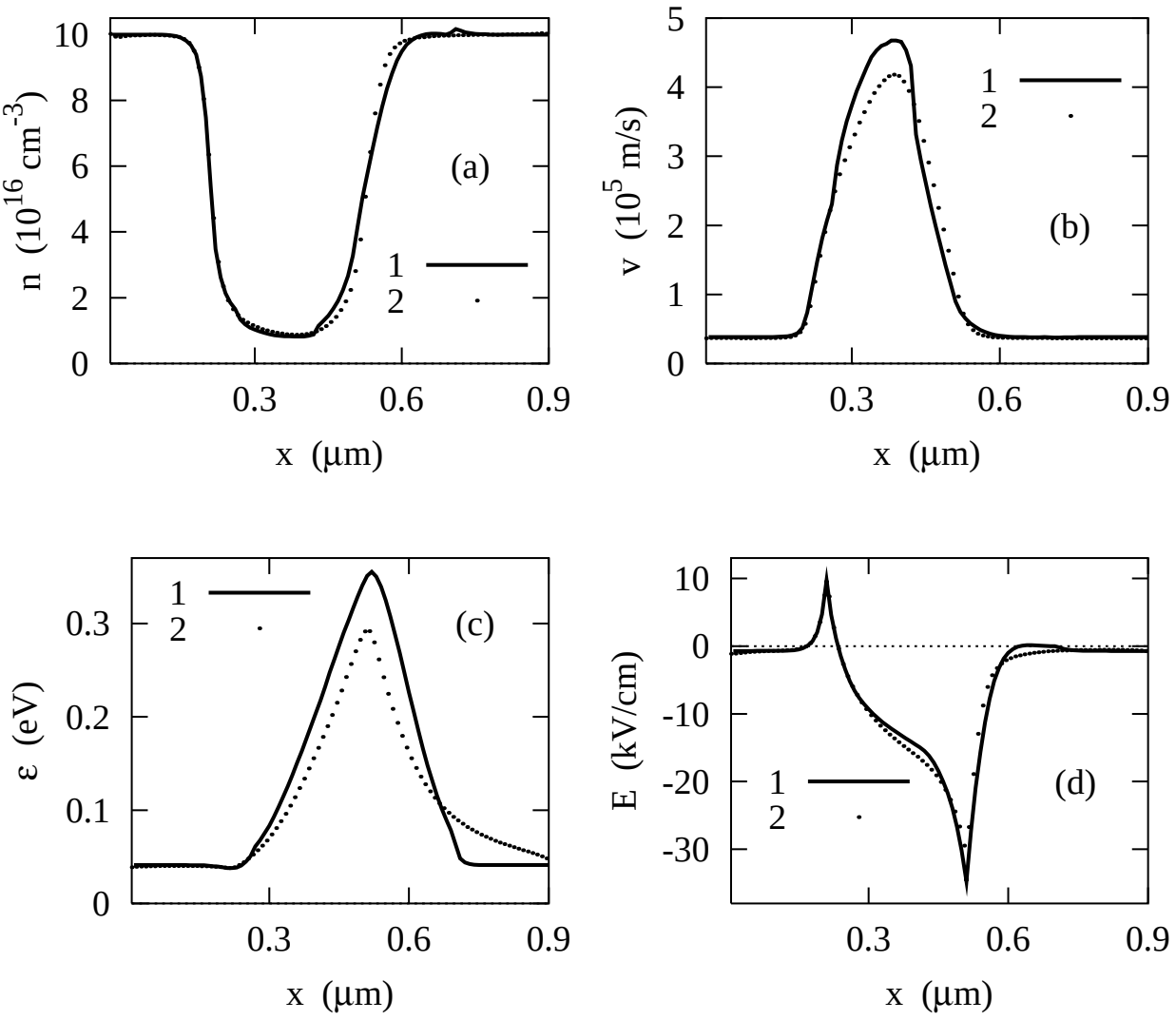

FIG. 2.

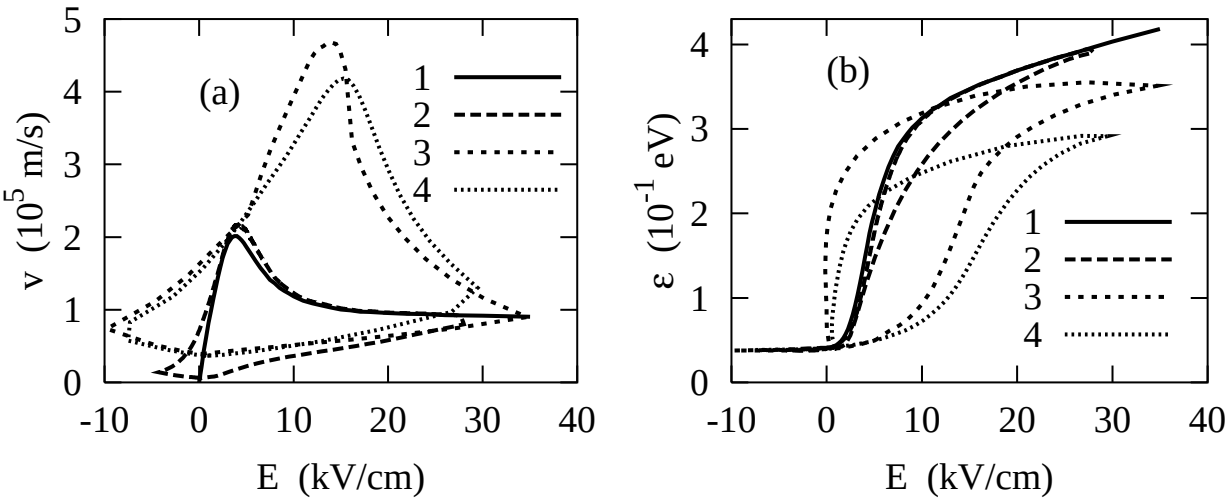

FIG. 3.

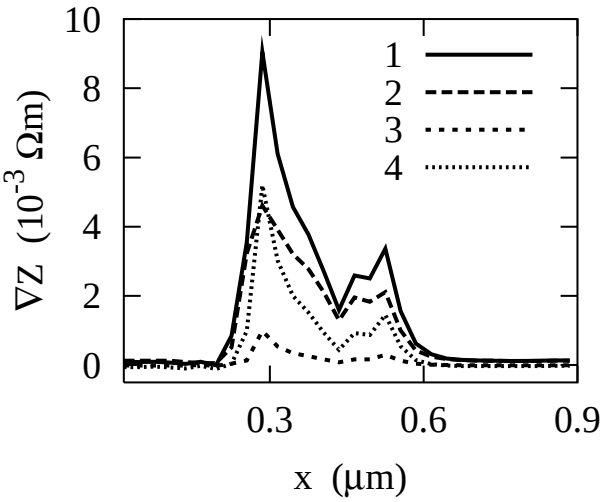

FIG. 4.

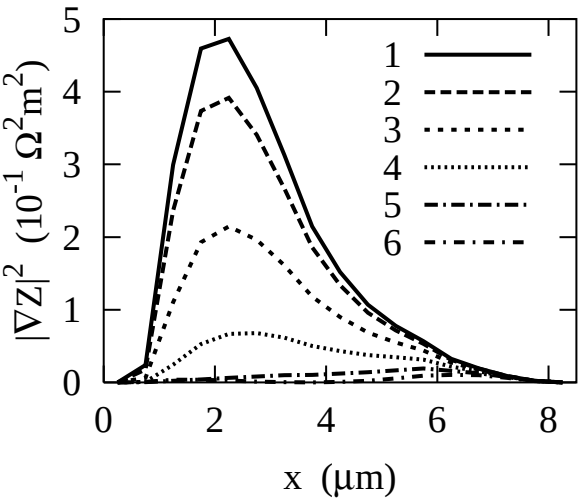

FIG. 5.

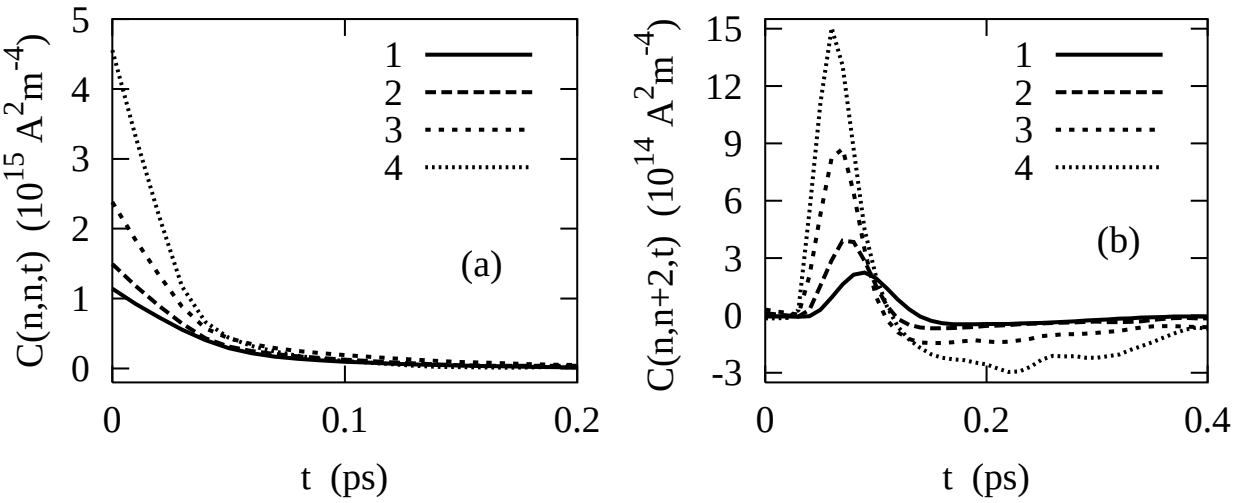
C(n,n,t) (10^15 A^2m^-4)
5
4
3
2
1
0
0
0.1
0.2
t (ps)
1
2
3
4
(a)
C(n,n+2,t) (10^14 A^2m^-4)
15
12
9
6
3
0
-3
0
0.2
0.4
t (ps)
1
2
3
4
(b)

FIG. 6.

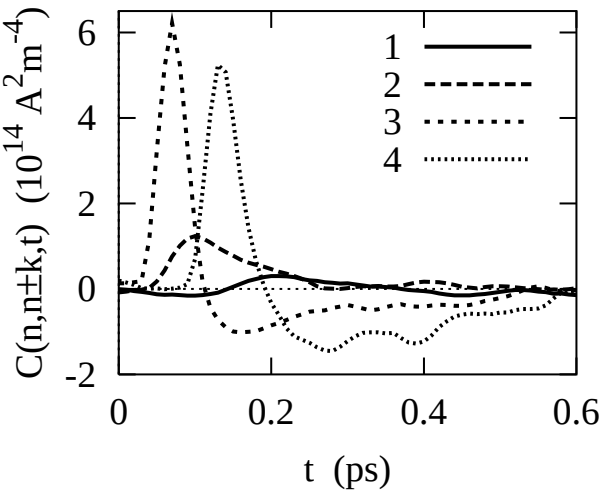

FIG. 7.

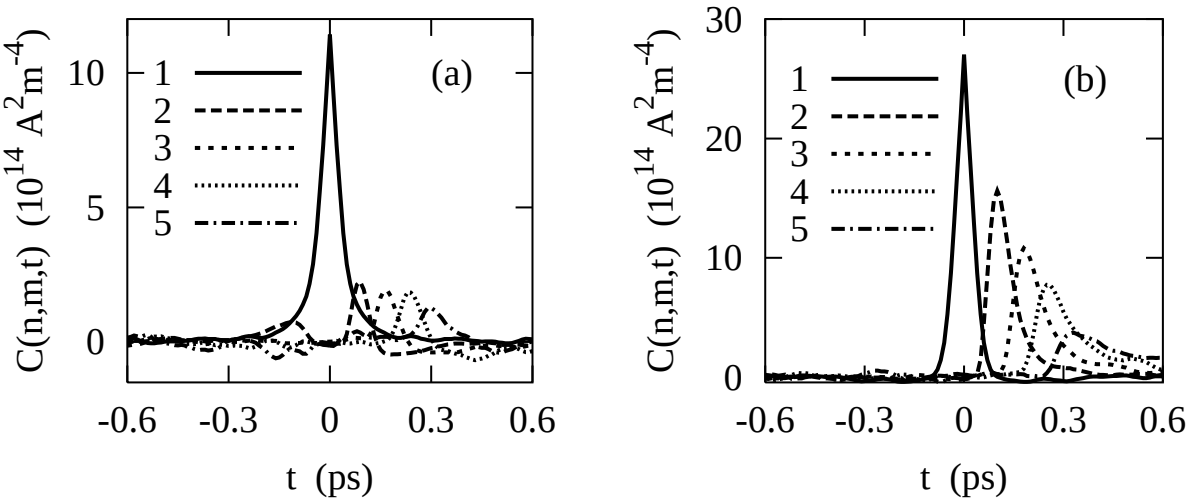

FIG. 8.

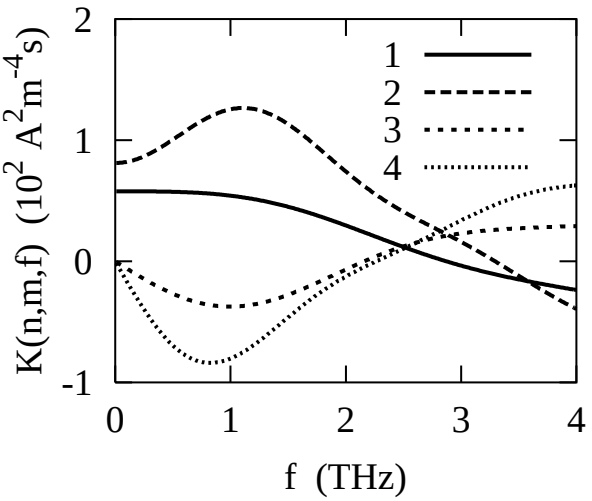

FIG. 9.

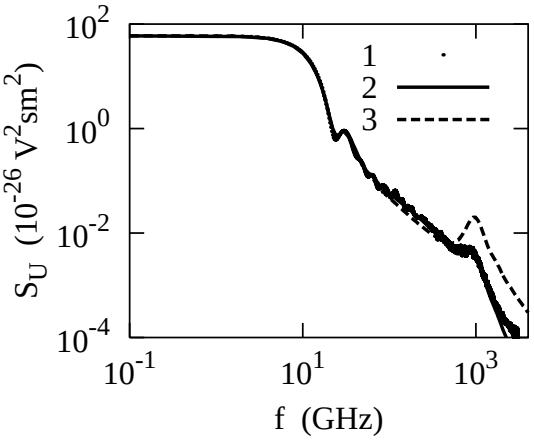

FIG. 10.

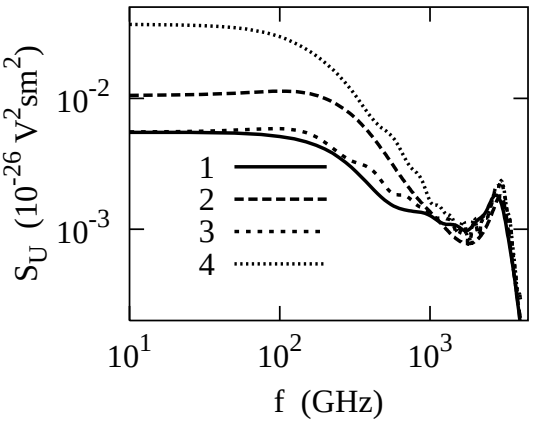

FIG. 11.

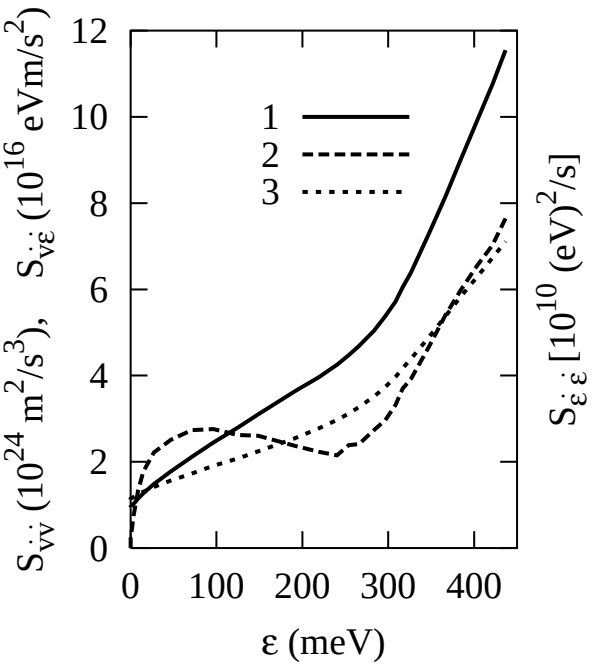
12
10
8
6
4
2
0
0
100
200
300
400
ε (meV)
1
2
3
$S_{\dot{v}\dot{v}}$ (10^{24} m^2/s^3), $S_{\dot{v}\dot{\varepsilon}}$ (10^{16} eVm/s^2)
$S_{\dot{\varepsilon}\dot{\varepsilon}}$ [10^{10} $(eV)^2$/s]

FIG. 12.

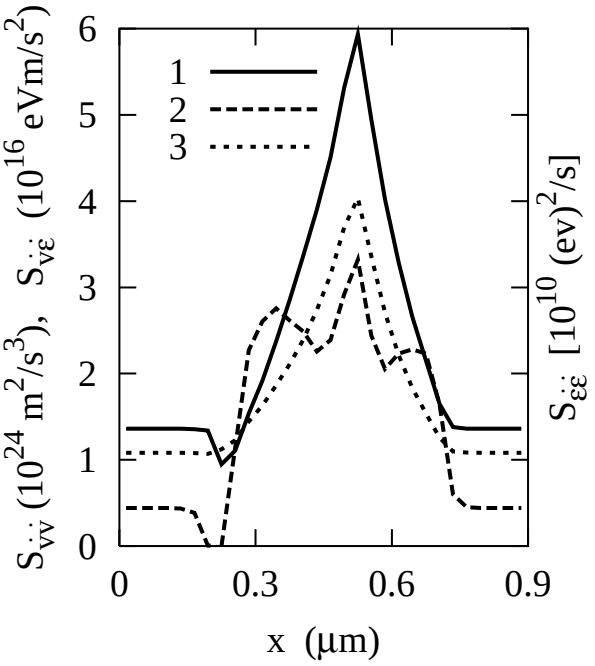
1
2
3
0
1
2
3
4
5
6
0
0.3
0.6
0.9
x (μm)
$S_{\dot{v}\dot{v}}$ (10^{24} m^2/s^3), $S_{\dot{v}\dot{\varepsilon}}$ (10^{16} eVm/s^2)
$S_{\dot{\varepsilon}\dot{\varepsilon}}$ [10^{10} (ev)^2/s]

FIG. 13.

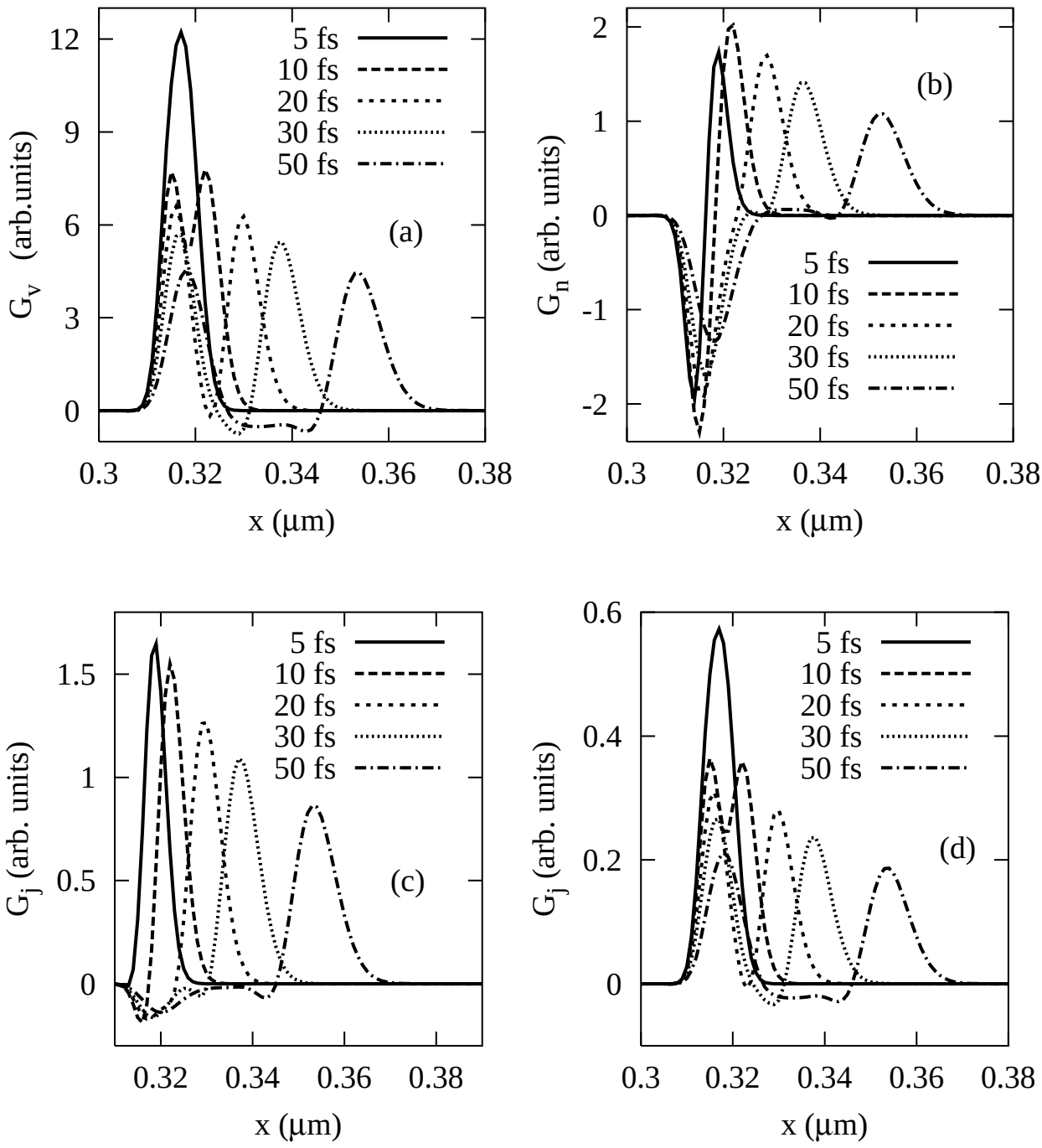
(a)
(b)
(c)
(d)
5 fs
10 fs
20 fs
30 fs
50 fs
G_v (arb.units)
G_n (arb. units)
G_j (arb. units)
x (μm)

FIG. 14.

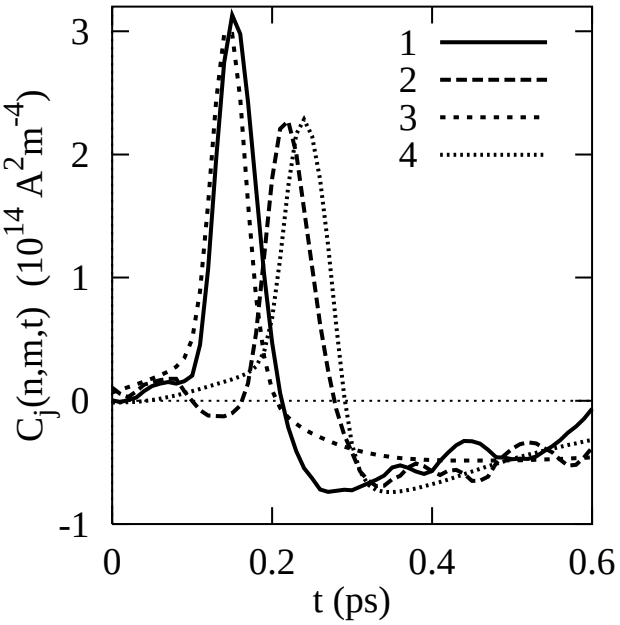

FIG. 15.

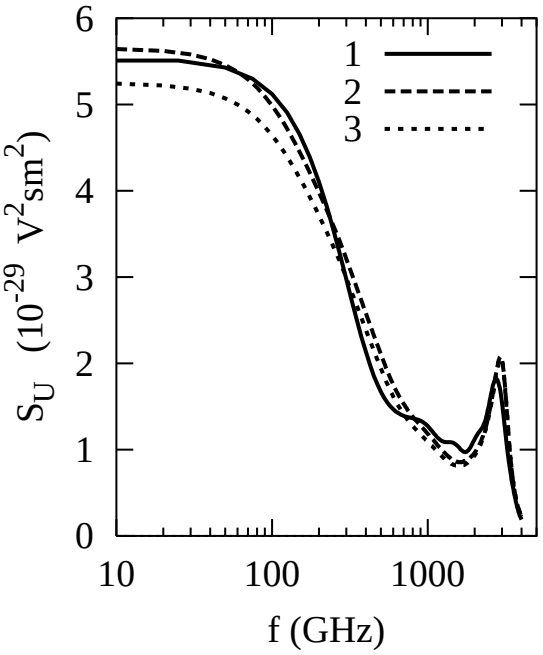

FIG. 16.

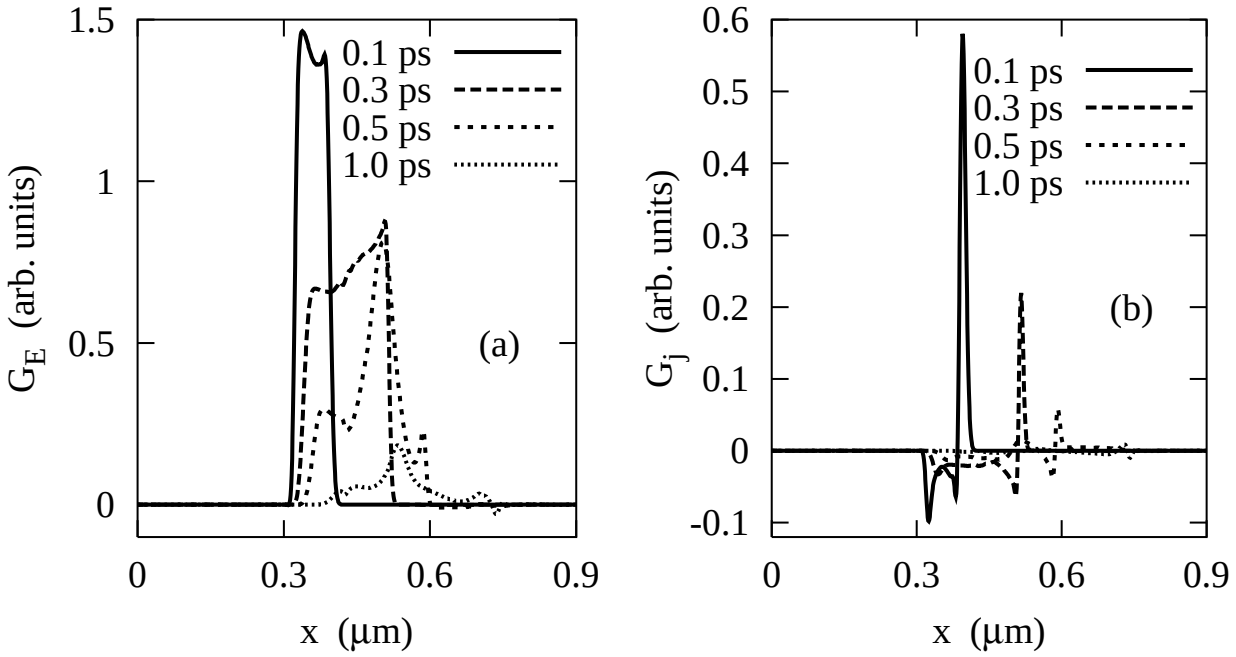
1.5
1
0.5
0
G_E (arb. units)
0.1 ps
0.3 ps
0.5 ps
1.0 ps
(a)
0
0.3
0.6
0.9
x (μm)
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
G_j (arb. units)
0.1 ps
0.3 ps
0.5 ps
1.0 ps
(b)
0
0.3
0.6
0.9
x (μm)

FIG. 17.

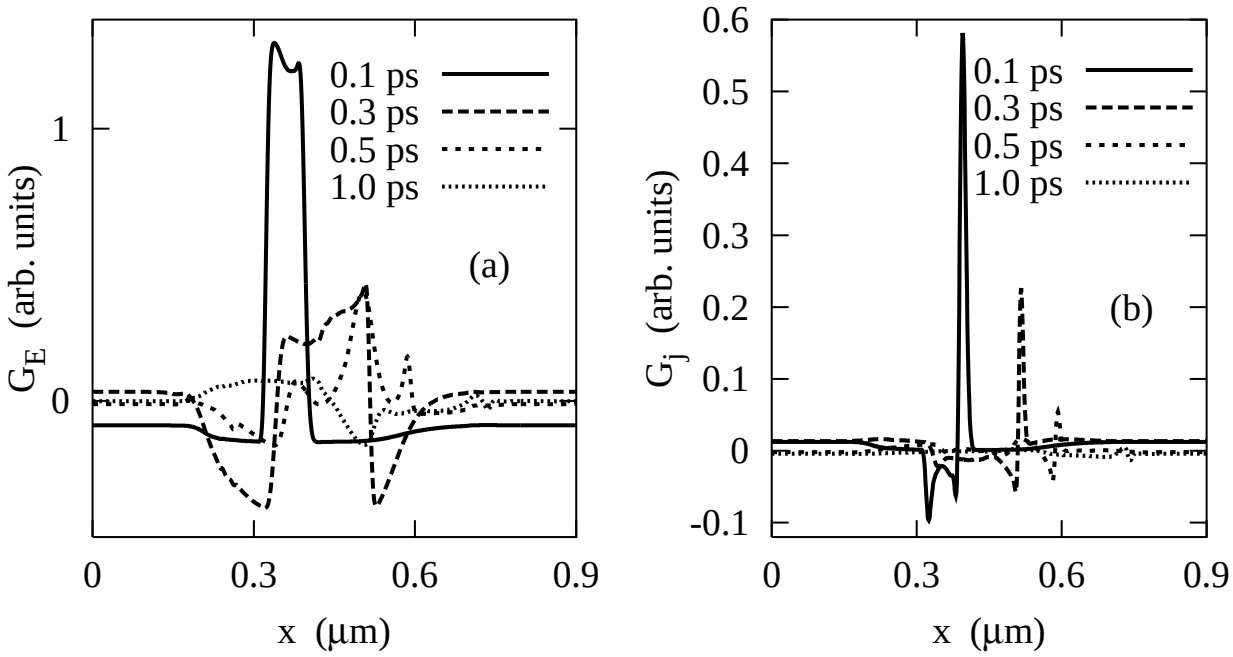

FIG. 18.

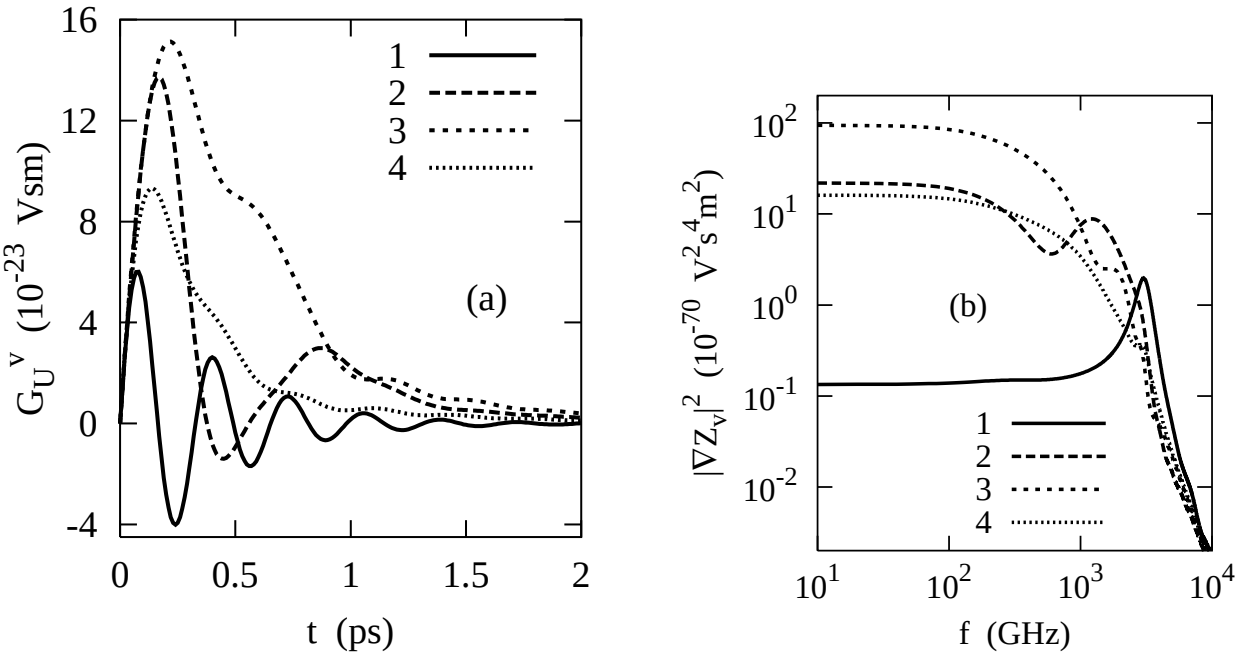
16
12
8
4
0
-4
0
0.5
1
1.5
2
t (ps)
G_U^v (10^{-23} Vsm)
1
2
3
4
(a)
10^2
10^1
10^0
10^{-1}
10^{-2}
10^1
10^2
10^3
10^4
f (GHz)
$|\nabla Z_v|^2$ (10^{-70} $V^2s^4m^2$)
(b)

FIG. 19.

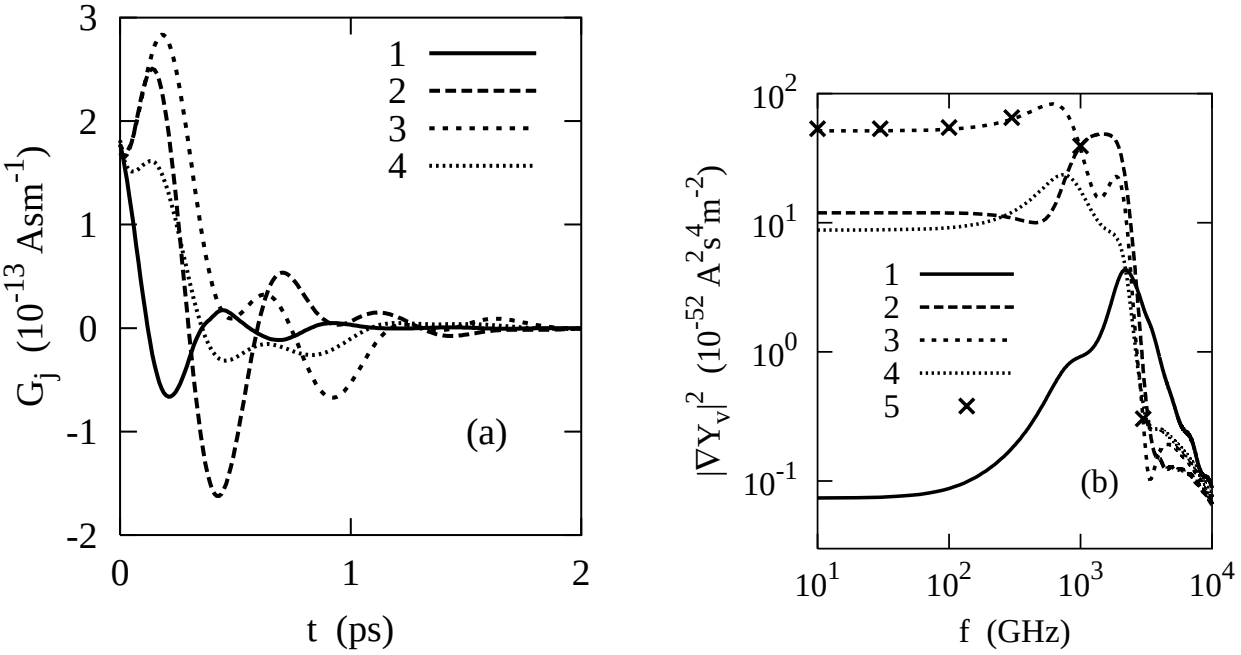
3
2
1
0
-1
-2
0
1
2
t (ps)
G_j (10^-13 Asm^-1)
1
2
3
4
(a)
10^2
10^1
10^0
10^-1
10^1
10^2
10^3
10^4
f (GHz)
|∇Y_v|^2 (10^-52 A^2s^4m^-2)
1
2
3
4
5
(b)

FIG. 20.

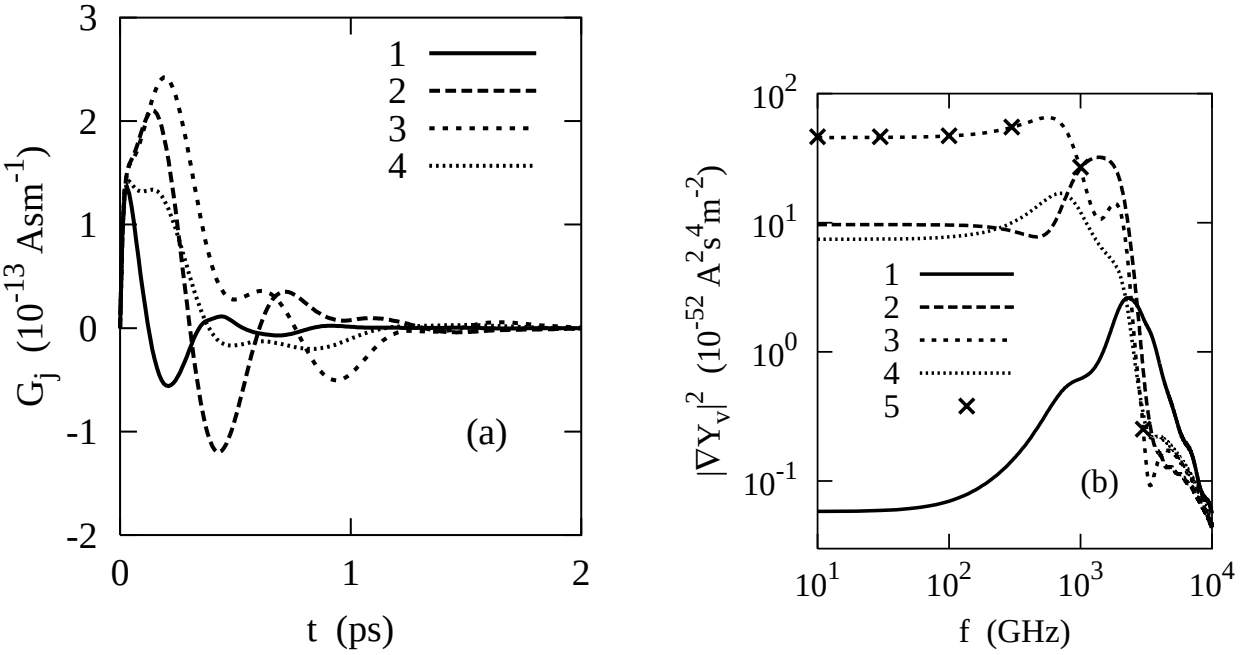
3
2
1
0
-1
-2
0
1
2
t (ps)
G_j (10^-13 Asm^-1)
1
2
3
4
(a)
10^2
10^1
10^0
10^-1
10^1
10^2
10^3
10^4
f (GHz)
|∇Y_v|^2 (10^-52 A^2s^4m^-2)
1
2
3
4
5
(b)

FIG. 21.

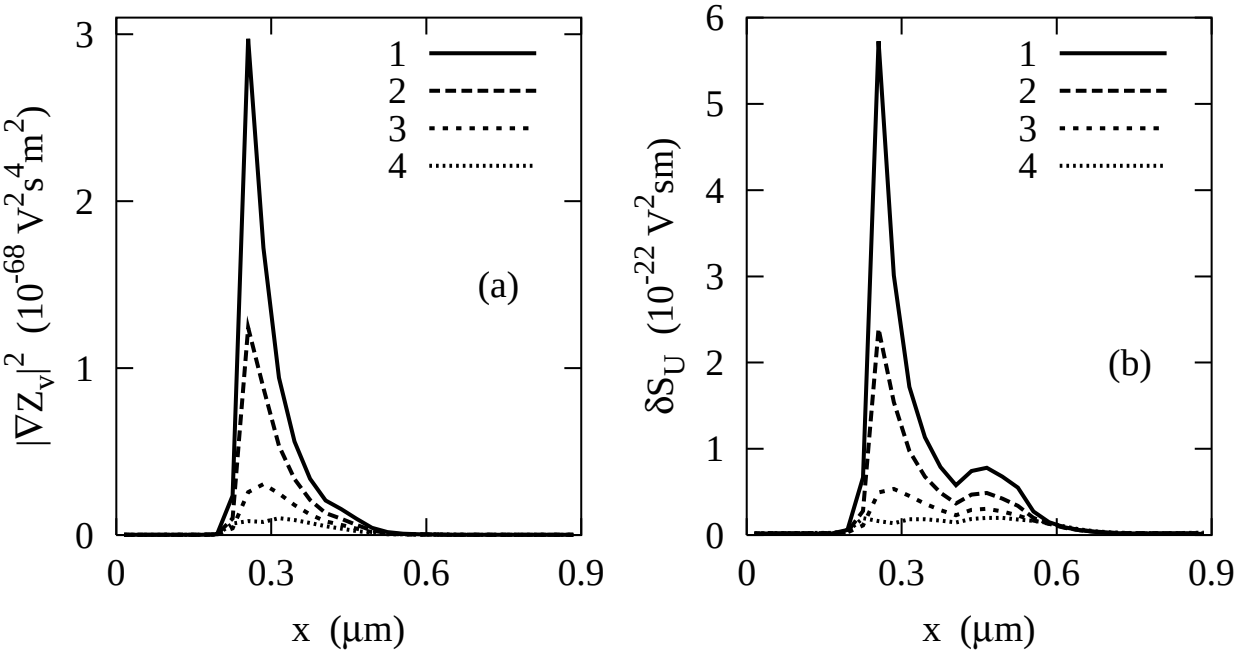
3
2
1
0
0
0.3
0.6
0.9
x (μm)
$|\nabla z_v|^2$ (10^{-68} $V^2s^4m^{-2}$)
1
2
3
4
(a)
6
5
4
3
2
1
0
δS_U (10^{-22} V^2sm)
(b)

FIG. 22.

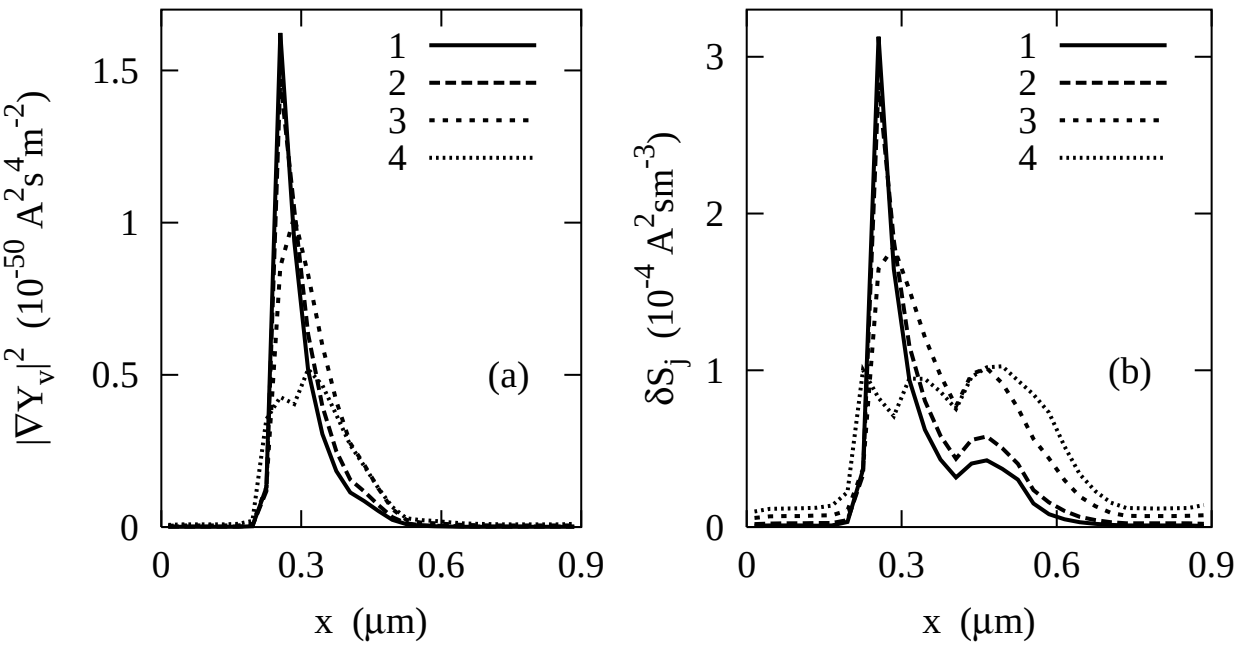
1.5
1
0.5
0
0
0.3
0.6
0.9
x (μm)
|∇Yv|2 (10-50 A2s4m-2)
(a)
1
2
3
4
3
2
1
0
δSj (10-4 A2sm-3)
(b)

FIG. 23.

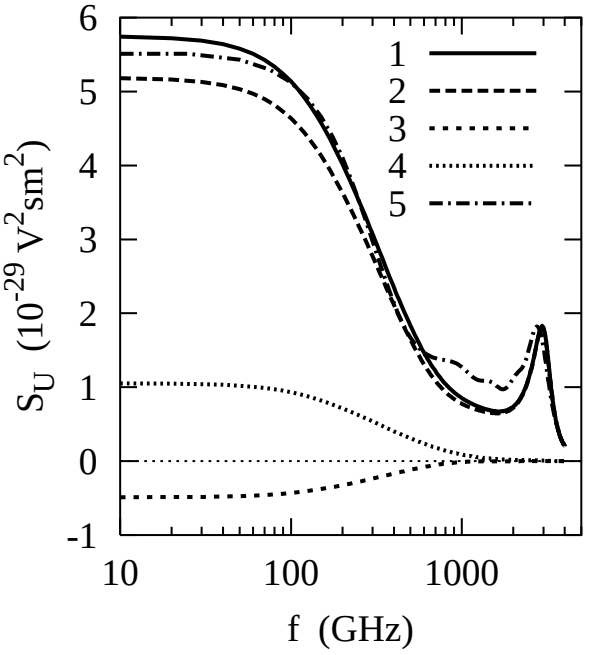

FIG. 24.

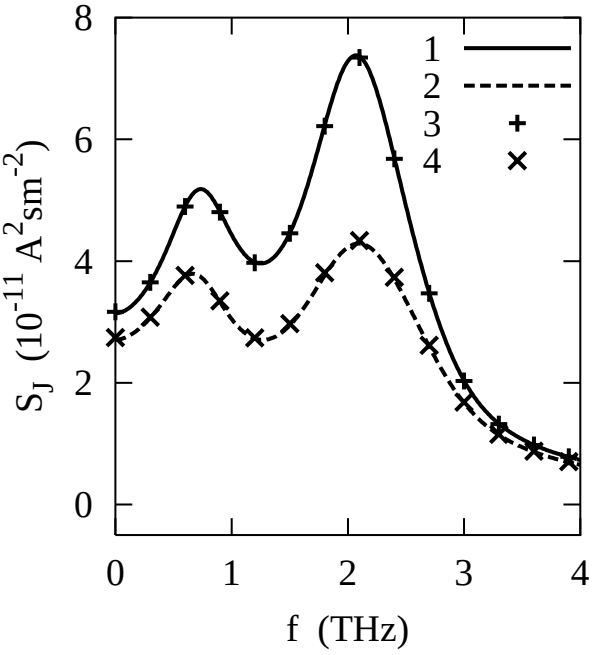

FIG. 25.

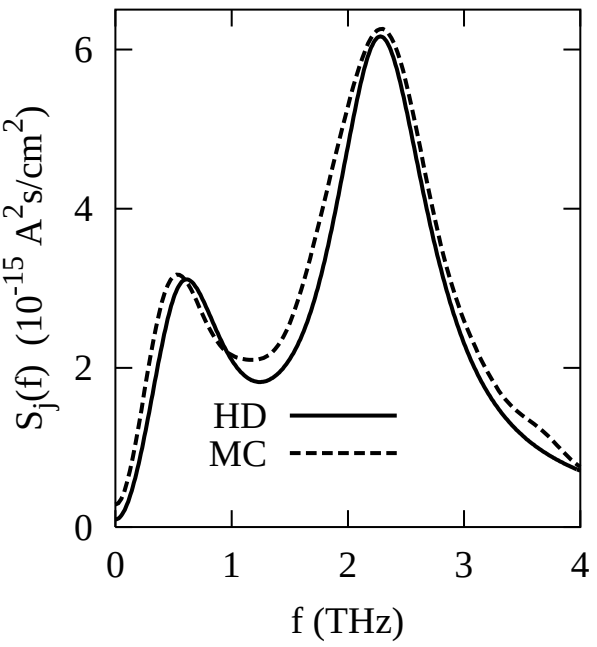

FIG. 26.

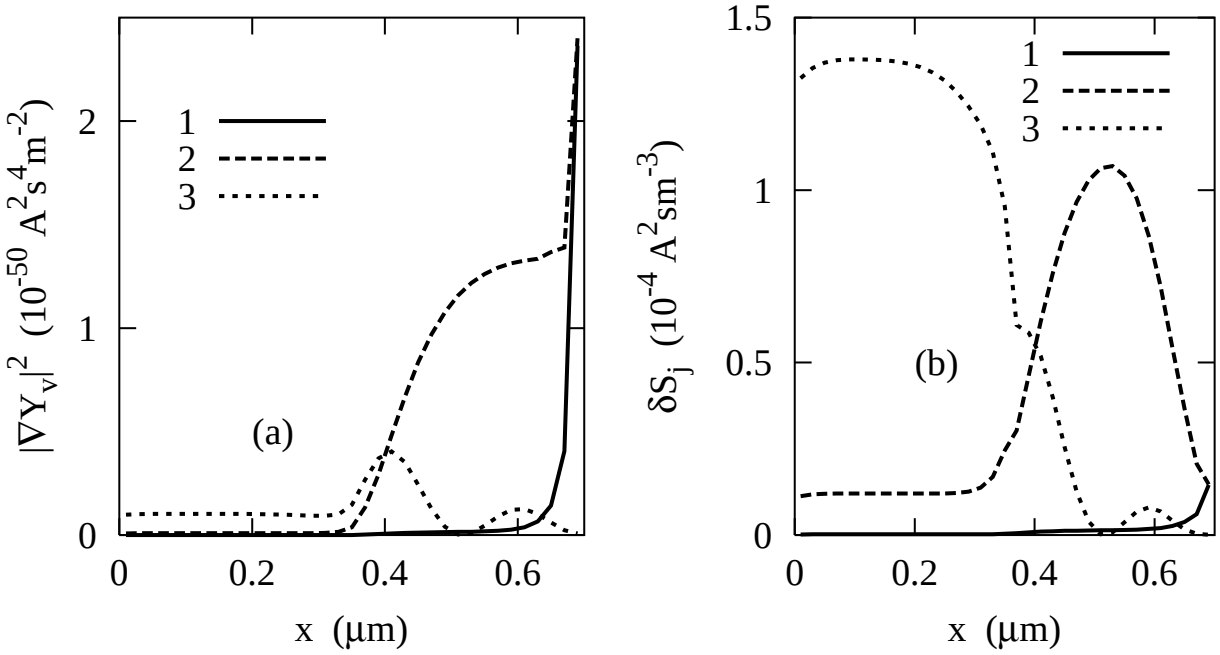
1
2
3
(a)
(b)
$|\nabla Y_v|^2$ (10^{-50} $A^2s^4m^{-2}$)
δS_j (10^{-4} A^2sm^{-3})
x (μm)
x (μm)
0
0.2
0.4
0.6
0
1
2
0
0.5
1
1.5

FIG. 27.

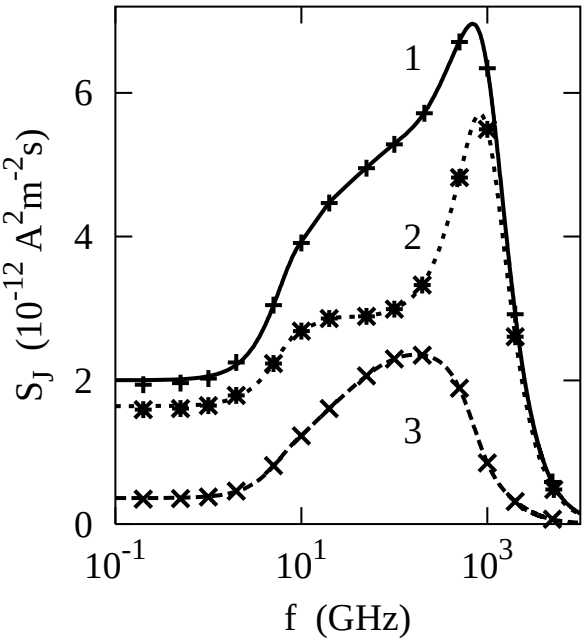

FIG. 28.

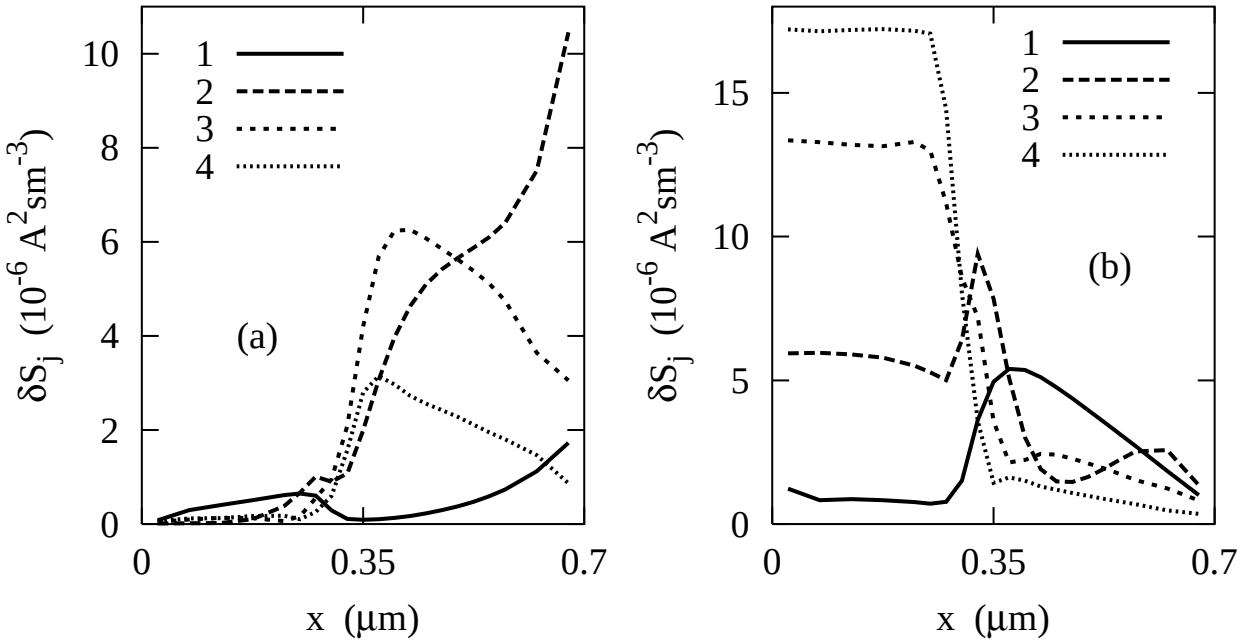
10
8
6
4
2
0
0
0.35
0.7
x (μm)
δS_j (10^-6 A^2sm^-3)
1
2
3
4
(a)
15
10
5
0
0
0.35
0.7
x (μm)
δS_j (10^-6 A^2sm^-3)
1
2
3
4
(b)

FIG. 29.

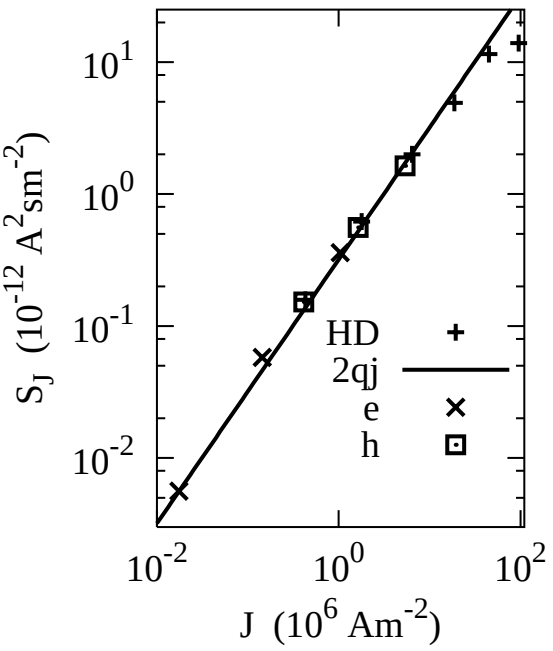

FIG. 30.