

Hydrodynamic Modeling of Spatial Cross-Correlation of Conduction Current Fluctuations

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Abstract. In the framework of the Green-function formalism the hydrodynamic-Langevin scheme is developed for electronic noise calculations by using deterministic conservation equations. The validity of the scheme is illustrated by calculating the cross-correlation functions of conduction current fluctuations in submicron GaAs structures.

Introduction. Within the Boltzmann transport equation (BTE) the most comprehensive description of the spatio-temporal evolution of fluctuations is usually based on the Green-function formalism, when the source of fluctuations is represented by the Langevin force describing the occupation number fluctuations caused by single scattering events [1,2]. However, because of its mathematical complexity, the BTE approach meets with serious difficulties when applied to calculate the transport and noise characteristics of real devices in the presence of hot-carrier conditions. As a consequence, deterministic approaches at a hydrodynamic (HD) level are better suited for this sake [3,4]. The aim of this work is to construct a similar formalism at a HD level keeping the main features of the BTE Langevin force determined by single scattering events. We notice that the HD approach is intrinsically based on a separate description of carrier concentration from dynamic characteristics such as velocity, energy, etc. Therefore, the two tasks to be solved are: (i) to reformulate the Langevin forces of the BTE in terms of fluctuations of the HD variables at the HD level, (ii) to construct appropriate response functions to the HD Langevin forces.

Hydrodynamic equations with Langevin forces. The formal procedure to incorporate the Langevin forces at a HD level is the same as for the derivation of the HD equations. By multiplying the BTE with the Langevin force $\xi(\mathbf{p}, x, t)$ by 1, $v(\mathbf{p})$, $\varepsilon(\mathbf{p})$ and further integration over momentum space one obtains the conservation equations for carrier concentration $n(x, t)$, mean velocity $v(x, t)$ and mean energy $\varepsilon(x, t)$:

$$\frac{\partial n}{\partial t} + \frac{\partial nv}{\partial x} = 0 \quad (1)$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \frac{1}{n} \frac{\partial}{\partial x} (nQ_v) - eEm^{-1} + v\nu_v = \frac{1}{n} \xi_v(x, t) \quad (2)$$

$$\frac{\partial \varepsilon}{\partial t} + v \frac{\partial \varepsilon}{\partial x} + \frac{1}{n} \frac{\partial}{\partial x} (nQ_\varepsilon) - eEv + (\varepsilon - \varepsilon_{th})\nu_\varepsilon = \frac{1}{n} \xi_\varepsilon(x, t) \quad (3)$$

where

$$\xi_\alpha(x, t) = \int \alpha(\mathbf{p}) \xi(\mathbf{p}, x, t) d\mathbf{p} \quad (4)$$

is the HD analog of the BTE Langevin force $\xi(\mathbf{p}, x, t)$. In the absence of generation-recombination processes, as it is the case considered here, the continuity equation (1) does not contain source of fluctuations, since during a scattering event a carrier keeps the same position in the real space, so that $\int \xi(\mathbf{p}, x, t) d\mathbf{p} = 0$ [1,2]. Therefore, the HD Langevin forces given by Eq. (4) describe fluctuations of only carrier velocity ($\alpha = v$) and energy ($\alpha = \varepsilon$), which can change during a scattering event. Eq. (4) represents the total intensity of the random force which acts on all carriers located in a small neighborhood around point x . Since Eqs. (2) and (3) are written for mean values of velocity and energy of one carrier, the quantity $\xi_\alpha(x, t)/n$ in the r.h.s. of Eqs. (2) and (3) corresponds to the intensity of the HD Langevin force normalized to one carrier.

Spectral power of HD Langevin forces. Similarly to the case of the BTE approach, the correlation function of the HD Langevin forces is δ -correlated both in time and space:

$$\overline{\xi_\alpha(x_0, t)\xi_\beta(x'_0, t+s)} = \frac{1}{2}S_{\alpha\beta}(x_0)\delta(x_0 - x'_0)\delta(s) \quad (5)$$

where $S_{\alpha\beta}(x_0)$ is the local spectral density of the fluctuation power of the HD Langevin forces ($\alpha = \beta$) and their cross-correlations ($\alpha \neq \beta$), which can be expressed through the correspondent spectral density $S_\xi(\mathbf{p}, \mathbf{p}', x_0)$ of the BTE Langevin forces by:

$$S_{\alpha\beta}(x_0) = \int d\mathbf{p} \int d\mathbf{p}' \alpha(\mathbf{p})\beta(\mathbf{p}')S_\xi(\mathbf{p}, \mathbf{p}', x_0) \quad (6)$$

Substitution into Eq. (6) of an analytical expression for $S_\xi(\mathbf{p}, \mathbf{p}', x_0)$ given in [1,2] for the case of nondegenerate carriers and in the absence of two-particle interaction, after some simple transformations finally gives:

$$S_{\alpha\beta}(x_0) = 2 \int d\mathbf{p} \int d\mathbf{p}' [\alpha(\mathbf{p}') - \alpha(\mathbf{p})][\beta(\mathbf{p}') - \beta(\mathbf{p})]W(\mathbf{p}, \mathbf{p}')f_s(\mathbf{p}, x_0) \quad (7)$$

where $f_s(\mathbf{p}, x_0)$ is the stationary distribution function of carriers in the structure normalized to carrier concentration, and $W(\mathbf{p}, \mathbf{p}')$ is the transition rate from state \mathbf{p} into state \mathbf{p}' due to all scattering mechanisms. As follows from Eq. (7) the power of the HD noise source is distributed in the volume proportionally to the carrier concentration $n_s(x_0) = \int f_s(\mathbf{p}, x_0) d\mathbf{p}$. Since in the HD approach the concentration is described by the separate equation (1) which does not contain Langevin forces, it is convenient to introduce the single-particle spectral densities $\tilde{S}_{\alpha\beta}(x_0) = S_{\alpha\beta}(x_0)/n_s(x_0)$, which local values directly depend on the single-carrier distribution function in the whole momentum space in the neighborhood of point $x = x_0$ only. The value of $\tilde{S}_{\alpha\beta}(x_0)$ is determined only by processes occurring during a single scattering event and is described by average rate of the velocity and energy changes, $\Delta_\alpha = \alpha(\mathbf{p}') - \alpha(\mathbf{p})$, which take place when the carrier instantaneously scatters from an initial state \mathbf{p} into a final state \mathbf{p}' . Since for most mechanisms the scattering rate depends on carrier energy only, it is reasonable to assume, as it was done for all other parameters of the HD approach, that also $\tilde{S}_{\alpha\beta}$ depends on the local value of the mean energy only. In this case numerical values of $\tilde{S}_{\alpha\beta}(\varepsilon)$, as functions of the mean energy ε can be obtained from MC simulations of the homogeneous bulk material as described in [4].

Hydrodynamic single-particle Green-functions. Since $S_{\alpha\beta}$ given by Eq. (7) is proportional to the local concentration $n_s(x_0)$ through $f_s(\mathbf{p}, x_0)$ we are allowed to introduce the concept of a single-particle Green-function which describes a *linear* response of characteristics of one particle to an action of the HD Langevin forces (4). For this sake the point-like impulsive forces which enter into the definition of correspondent Green-functions

must be locally normalized to an unperturbed concentration, i.e. they should be of the form $\delta(x - x_0)\delta(t)/n_s(x_0)$. It means, that the normalization factor $1/n$ in the r.h.s. of Eqs. (2) and (3) will be separated from $\xi_\alpha(x, t)$, and placed into the definition of the Green-function. Let us consider the general case of a numerical calculation of the Green-function (response function) for an arbitrary local characteristic of the system, $Q(n, v, \epsilon, E, x)$, which can be represented as a function of the HD variables (n, v, ϵ) , the local electric field E and position x inside the structure. The simplest examples of such a Q -characteristic are the HD variables themselves. The conduction current, $j_{cond} = env$, the energy flux, $W = env\epsilon$, etc. are other examples.

The procedure for the Green-functions calculation is as follows. Firstly, a stationary solution of Eqs. (1)-(3) without the Langevin forces is found together with the Poisson equation for the self-consistent electric field $E(x)$ and, if necessary, another equation describing the external circuit. Then, a perturbation of the stationary values of velocity or energy, given by $\Delta_\alpha\delta(x - x_0)$, is introduced at time $t = 0$ at some point x_0 . Usually, the spatial profile of the perturbation is given by an approximation of the δ -function which takes some volume in x -space, for example a Gaussian function. The perturbation amplitude Δ_α is taken in such a way to fulfill the requirement of the response linearity to the initial perturbation. Then, a direct numerical solution of the system of equations (1) - (3) jointly, if necessary, with the Poisson and circuit equations is performed thus providing the relaxation of the system to the stationary state at $t > 0$. The correspondent Green-function of a certain Q -characteristic is obtained from the difference between local values of $Q(x, x_0, t)$ calculated during the relaxation process and $Q_s(x)$ corresponding to stationary conditions which is then normalized to the amplitude of the initial perturbation Δ_α and the carrier concentration $n_s(x_0)$ at the point of the perturbation as:

$$G_q^\alpha(x, x_0, t) = \frac{1}{\Delta_\alpha n_s(x_0)} [Q(x, x_0, t) - Q_s(x)] \quad (8)$$

where $\alpha = v, \epsilon$ when the perturbation of velocity or energy are introduced, respectively.

In accordance with the Green-function formalism, a local fluctuation of a Q -characteristic caused by the HD Langevin force (4) can be represented as:

$$\delta q(x, t) = \sum_{\alpha=v,\epsilon} \int dx_0 \int_0^\infty ds G_q^\alpha(x, x_0, s) \xi_\alpha(x_0, t - s) \quad (9)$$

From Eq. (9) the spatio-temporal dependence of auto-correlation ($q = q'$) and cross-correlation ($q \neq q'$) functions of various Q -characteristics can be represented as:

$$C_{qq'}(x', x'', s) = \sum_{\alpha,\beta=v,\epsilon} \int dx_0 n_s(x_0) \tilde{S}_{\alpha\beta}(x_0) \int_0^\infty G_q^\alpha(x', x_0, u) G_{q'}^\beta(x'', x_0, u + s) du \quad (10)$$

As follows from Eq. (10), in full analogy with the stochastic MC interpretation of the BTE, the perturbations given by $\xi_\alpha(x_0, t)$ can be considered in terms of a sequence of instantaneous impulsive variations of probable values of velocity and energy of a single particle which further dynamics is governed by the HD conservation equations (1)-(3). Indeed, Eq. (10) is the convolution the following three factors. The local origin of particle fluctuations given by $\tilde{S}_{\alpha\beta}(x_0)$, its spatio-temporal evolution described by single-particle Green-functions and the carrier concentration. From the above interpretation we argue that multiplication by the carrier concentration and integration over the whole device is equivalent to averaging over an ensemble of independent carriers.

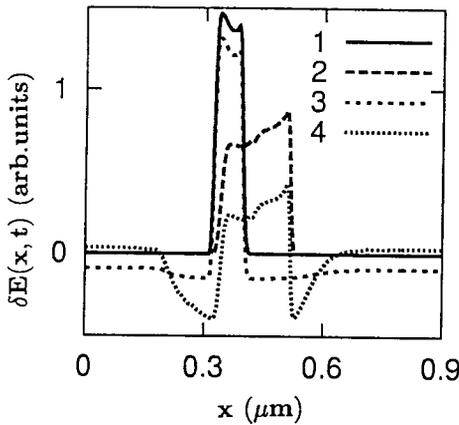


Fig. 1 - Electric field response at time moments $t = 0.1, 0.3 ps$ (curves 1,3 and 2,4, respectively) to a velocity perturbation placed at $t = 0$ at the point $x = 0.315 \mu m$ for the case of constant current (curves 1,2) and constant voltage (curves 3,4) operation.

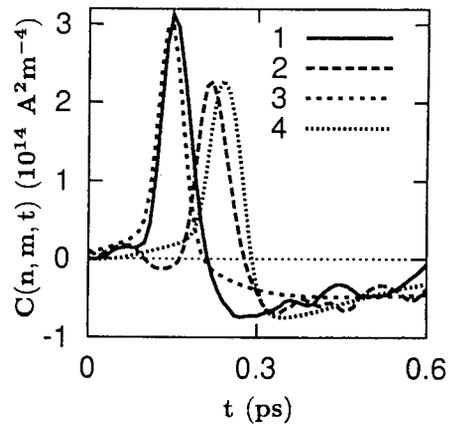


Fig. 2 - Cross-correlation functions $C(n, m, t)$ calculated by MC (curves 1,2) and HD (curves 3,4) methods for cells m centered at the points $x = 0.435, 0.495 \mu m$ (curves 1,3 and 2,4, respectively) and a cell n centered at $x = 0.315 \mu m$.

Numerical results. Numerical calculations are performed for a $0.21-0.30-0.39 \mu m n^+nn^+$ GaAs structure with doping levels $n = 5 \times 10^{15} cm^{-3}$ and $n^+ = 10^{17} cm^{-3}$ at $T = 300 K$ for a voltage of $0.5 V$. To illustrate the possibility of Green-functions calculations under various conditions, Fig. 1 presents the electric field response at time moments $t = 0.1, 0.3 ps$ (curves 1,3 and 2,4, respectively) to a velocity perturbation placed at $t = 0$ at the point $x = 0.315 \mu m$ for the case of constant current (curves 1,2) and constant voltage (curves 3,4) operation mode. Due to different operation modes, in the former case the response keeps the initial local character, while in the latter case the response covers the whole device at any time moment. Figure 2 compares the cross-correlation functions of conduction-current fluctuations associated with velocity fluctuations only (i.e. the HD calculations are performed for $Q = en_s(x)v(x)$ calculated under static field conditions by MC (curves 1 and 2) and HD (curves 3 and 4) methods. In accordance with [5] this is the noise source for the standard IF method when spatial correlation of the diffusion noise sources are accounted for. We consider the overall agreement between the results of the two methods satisfactory thus supporting the validity of the HD-Langevin scheme here developed.

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