Spatiotemporal correlation of conduction current fluctuations within a hydrodynamic-Langevin scheme

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We present a hydrodynamic-Langevin scheme to describe electronic noise in unipolar structures and evaluate the cross-correlation functions of conduction current fluctuations entering the transfer impedance method. The theory is developed in terms of microscopic fluctuations of carrier velocity and energy taking place during scattering events. Applications to submicron n^+nn^+ GaAs structures show that the contribution of stochastic velocity rate prevails over that of the energy rate in determining the cross-correlation function of conduction current fluctuations. © *1999 American Institute of Physics.* [S0003-6951(99)01105-5]

From the microscopic point of view, electronic noise in semiconductor devices is in essence originated by scattering events which carriers undergo with static lattice imperfections and phonons. In the framework of the Boltzman transport equation, the stochastic nature of these events is usually represented by Langevin forces δ correlated both in time and space.[1] These forces lead to fluctuations of the distribution function $\delta f(\mathbf{p},\mathbf{r},t)$, which, in turn, are responsible for fluctuations of physical quantities at a macroscopic level. Among them, the most important are fluctuations of the local conduction current defined as $\delta j_c(\mathbf{r},t) = e\int v(\mathbf{p})\,\delta f(\mathbf{p},\mathbf{r},t)\,d\mathbf{p}$. Under current operation mode, when a constant voltage is applied to a two-terminal device, these fluctuations lead to fluctuations of the total current as measured in the external circuit, $\delta J(t)=\int_0^L j_c(x,t)\,dx/L$, L being the length between the terminals. Under voltage operation mode, when a constant J is forced to flow through the device, these fluctuations lead to fluctuations of the voltage drop as measured between the terminals. At a macroscopic level, the latter operation mode is usually described by the impedance field (IF) method, which relates through the IF the voltage noise with local fluctuations of the conduction current (more precisely, with the contribution associated with velocity fluctuations only), represented by the so-called diffusion noise source. In over-micron devices, the noise source is well approximated by the local spectral density of velocity fluctuations.[2–4] In submicron devices, when characteristic lengths of macroscopic variations become comparable with the carrier mean-free path, the spatial correlation of the fluctuations becomes rather important[5,6] and, if neglected, a considerable overestimation of the low-frequency voltage noise has been found.[7] In this case, the noise source in the time domain should be more properly described by the two-point correlation function:

$$C(x',x'',s)=\overline{\delta j_c(x',t)\,\delta j_c(x'',t+s)}^{t}, \tag{1}$$

where the bar denotes the time average. The quantity in Eq. (1) can be directly calculated at a kinetic level by a Monte Carlo (MC) simulation[5,6] taking, however, a considerable amount of computer time.

The aim of this letter is to present a hydrodynamic (HD)-Langevin scheme able, on the one hand, to investigate the spatiotemporal evolution of conduction current fluctuations under different biasing conditions and, on the other hand, to evaluate their cross correlations with sufficient accuracy to be used within the transfer impedance method, thus avoiding the very lengthy MC simulation of the whole device. The scheme is developed by evaluating the first three moments of the Boltzmann–Langevin equation. For the conservation equations of carrier concentration $n(x,t)$, average velocity $v(x,t)$, and mean energy $\epsilon(x,t)$, it is obtained:

$$\frac{\partial n}{\partial t}=-\frac{\partial nv}{\partial x}+\xi_n, \tag{2}$$

$$\frac{\partial v}{\partial t}=eEm^{-1}-v\,\nu_v-v\,\frac{\partial v}{\partial x}-\frac{1}{n}\,\frac{\partial}{\partial x}(nQ_v)+\xi_v, \tag{3}$$

$$\frac{\partial \epsilon}{\partial t}=eEv-(\epsilon-\epsilon_{\rm th})\,\nu_\epsilon-v\,\frac{\partial \epsilon}{\partial x}-\frac{1}{n}\,\frac{\partial}{\partial x}(nQ_\epsilon)+\xi_\epsilon, \tag{4}$$

with the usual meaning of symbols and where $\xi_n(x,t)$, $\xi_v(x,t)$, and $\xi_\epsilon(x,t)$ are the corresponding random sources at a HD level. Let us note that, in general, the different sources are coupled since they reflect the same scattering event. In the following, we neglect generation–

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recombination processes and thus take $\xi_n = 0$. The set of Eqs. (2)–(4) is complemented by the Poisson equation:

$$\frac{dE(x,t)}{dx} = \frac{e}{\epsilon\epsilon_0}[n(x,t) - N_d(x)], \tag{5}$$

where $N_d(x)$ is the donor concentration profile.

Without the random sources, Eqs. (2)–(5) form a closed system of deterministic equations suitable for the numerical modeling of unipolar carrier transport in semiconductor devices under current operation mode.[8] In the opposite case of voltage operation mode an additional equation for the voltage drop between the structure terminals $U(t)$ must be added:[8]

$$\frac{d}{dt}U(t) = -\frac{L}{\epsilon\epsilon_0}\left[J - \frac{1}{L}\int_0^L j_c(x,t)dx\right], \tag{6}$$

where $j_c(x,t) = en(x,t)v(x,t)$ is the conduction current component along the structure. To describe satisfactorily the transport through the boundaries of an active region (n region) sandwiched between two contacts, additional n^+ regions (acting as Ohmic contacts) with $n^+ \gg n$ are usually introduced into the modeling of a whole n^+nn^+ structure. This allows one to take into account in a natural way the physical processes occurring near the n-region boundaries such as, redistribution of carrier concentration and thermalization of fluxes outcoming from the active region, etc. Within this prescription, the HD model described above yields the stationary profiles of concentration, velocity, and energy for given values of U or J. Furthermore, the same approach has been proved to describe satisfactorily both static and dynamic characteristics of the whole structure even when inside the active n-region carrier ballistic transport occurs at the characteristic scales of the order of, or even less than, the carrier mean-free path.[7–10]

To account for fluctuations associated with single scattering events occurring at point x_0 and time t_0 of the device under test, the random impulsive force for such an event can be represented as

$$\xi_\alpha(x_0,t_0) = \Delta_\alpha \delta(x - x_0)\,\delta(t - t_0), \tag{7}$$

where Δ_α is an appropriate amplitude for velocity ($\alpha = v$) or energy ($\alpha = \epsilon$) stochastic rates undergone by carriers during scatterings. Substitution of Eq. (7) into Eqs. (3) and (4) provides, after time integration, the initial conditions at $t = t_0$ of the corresponding perturbations. Therefore, starting from the stationary profiles and superimposing a fluctuation of velocity or energy described by the impulsive force (7), a numerical solution of the deterministic model allows one to study the spatiotemporal evolution of a single fluctuation of velocity or energy as well as the induced perturbations of concentration, field, conduction current, etc., under both operation modes. This procedure is applied to a 0.21–0.3–0.39 μm n^+nn^+ GaAs structure with doping levels $n = 5 \times 10^{15}\,\mathrm{cm}^{-3}$ and $n^+ = 10^{17}\,\mathrm{cm}^{-3}$ at $T = 300\,\mathrm{K}$ and $U = 0.5\,\mathrm{V}$. Uncertainty of numerical results are estimated within 1% at worst by an appropriate choice of the space–time grids.

Figure 1 presents the results for the time response of the conduction current in points $x = 0.315$, 0.375, 0.435, and 0.495 μm when an initial perturbation of velocity and energy

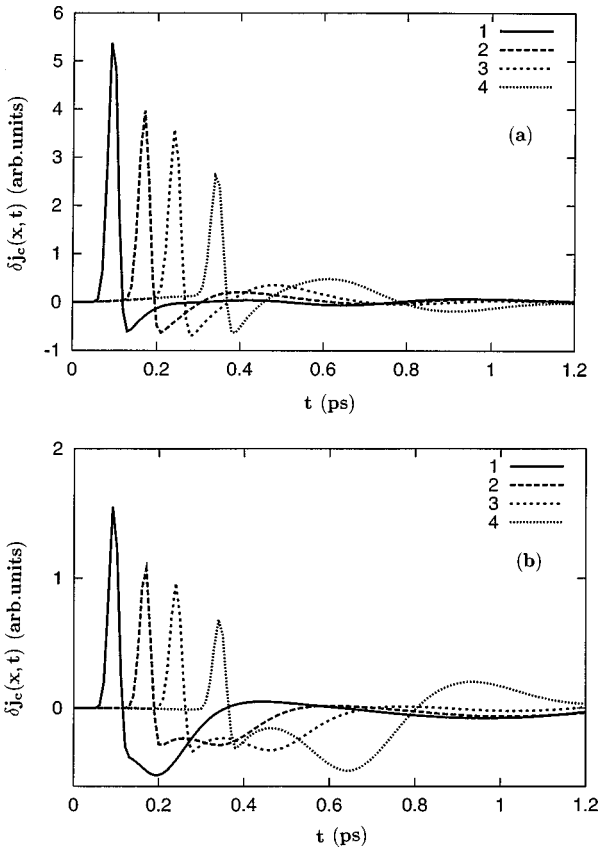

FIG. 1. Conduction current response calculated under self-consistent field conditions by the HD approach in points $x = 0.315$, 0.375, 0.435, and 0.495 μm, curves 1–4, respectively, for single (a) velocity and (b) energy fluctuations, placed at $x_0 = 0.255\,\mu$m. Calculations refer to a 0.21–0.3–0.39 μm n^+nn^+ GaAs structure with doping levels $n = 5 \times 10^{15}\,\mathrm{cm}^{-3}$ and $n^+ = 10^{17}\,\mathrm{cm}^{-3}$ at $T = 300\,\mathrm{K}$ with $U = 0.5\,\mathrm{V}$.

[Figs. 1(a) and 1(b), respectively] is placed at point $x_0 = 0.255\,\mu$m. Calculations are performed for the general case of a constant voltage applied between the structure terminals when the perturbations of the induced self-consistent electric field are accounted for. Each response curve shows a certain delay time, which is necessary for the right wing of the perturbation to cross the distance $x - x_0$. Then, the response exhibits a maximum when the main part of the perturbation crosses point x. The further relaxation to zero of the local response occurs on a time scale of about 1–2 ps.

After a proper normalization, the time responses can provide the conduction current components of the Green-function matrix associated with a single-carrier fluctuation at time $t_0 = 0$, $G_\alpha^j(x,x_0,t) = \delta j_c(x,x_0,t)/[n(x_0)\Delta_\alpha]$. For this sake, the amplitudes Δ_α must be taken to be sufficiently small to guarantee the linearity of response. In accordance with linear theory, the response of the local conduction current to the velocity and energy rates due to all scattering events at all previous times in the whole structure is

$$\delta j_c(x,t) = \sum_\alpha \int_0^L dx_0 \int_0^\infty dt' G_\alpha^j(x,x_0,t')\xi_\alpha(x_0,t-t'). \tag{8}$$

Substituting Eq. (8) into Eq. (1), and accounting for the δ-like correlation of the random sources, it becomes

$$C(x',x'',s)=\sum_{\alpha,\beta}\int_0^L dx_0 n(x_0) S_{\dot{\alpha}\dot{\beta}}[\epsilon(x_0)]$$

$$\int_0^\infty du G_\alpha^j(x',x_0,u) G_\beta^j(x'',x_0,u+s), \tag{9}$$

where $S_{\dot{\alpha}\dot{\beta}}(\epsilon)=\overline{\Delta_\alpha \Delta_\beta}^t$ are the microscopic noise sources corresponding to velocity and energy stochastic rates during scatterings. These are now supposed to depend on the local mean-energy $\epsilon(x_0)$, and as such can be easily calculated, together with the other parameters needed by the HD approach, from a MC simulation of the bulk material as described in Ref. 9.

To illustrate the approach developed above, we will show the HD calculation of the diffusion noise source to be used in the impedance field method when the spatial correlations are taken into account. In accordance with Ref. 7 such a source is represented in the time domain by the correlation function of conduction current fluctuations associated only with velocity fluctuations, $\delta j_c^v(x,t)=en(x)\delta v(x,t)$, and calculated under static field conditions. For this case one must replace G_α^j in Eqs. (8) and (9) by $en(x)\delta v(x,x_0,t)/[n(x_0)\Delta_\alpha]$. The values of $S_{\dot{\alpha}\dot{\beta}}(\epsilon)$, necessary to perform calculations in accordance with Eq. (9) are taken from Ref. 9.

To evaluate the spatial cross correlation of $\delta j_c^v(x,t)$, the n^+nn^+ structure is subdivided into cells of 300 Å each and the cross-correlation function $C(n,m,t)$ for $\delta j_c^v(x,t)$ averaged over cells n and m is calculated. The results obtained from HD and MC calculations for cells m centered at points $x=0.315$ and 0.375 μm and cell n centered at $x=0.255$ μm are compared in Fig. 2. The main features of $C(n,m,t)$, such as delay times, position of peaks, etc., obtained from both calculations are quite similar. The HD peaks are slightly shifted to shorter times due to some overestimation of velocity overshoot in HD modeling with respect to MC simulation of the same structure. As follows from Fig. 2, the main contribution to the total $C(n,m,t)$ (HD curves 3 and 4) comes from the $S_{\dot{v}\dot{v}}$ term (HD curves 5 and 6).

In conclusion, we have shown that the cross-correlation functions of conduction current fluctuations in a given structure can be described in the framework of a HD approach by means of the local current response to the microscopic velocity and energy δ-like fluctuations associated with scattering events. We stress that the great advantage of this scheme is that the input parameters needed to evaluate the spatial correlations are easily calculated from very simple bulk MC simulations. In this way, the direct but lengthy and unconvenient MC simulation of the whole device under analysis for the *in situ* evaluation of the spatial correlations[7] is avoided completely.

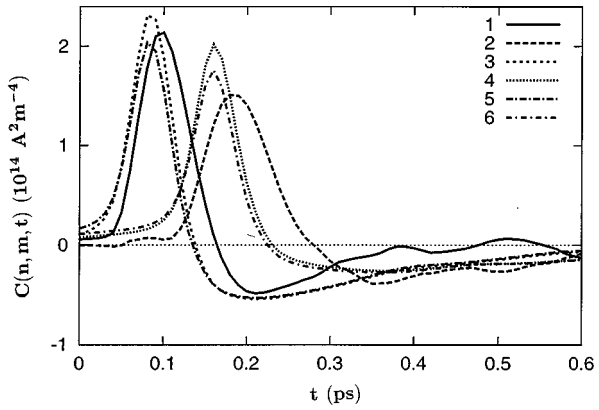

FIG. 2. Cross-correlation functions of conduction current fluctuations associated with velocity fluctuations only $C(n,m,t)$ calculated under static field conditions by MC (curves 1 and 2) and HD (curves 3–6) methods for cells m centered at the points $x=0.315$ and 0.375 μm (curves 1,3,5 and 2,4,6, respectively) and a cell n centered at $x=0.255$ μm in the same structure of Fig. 1. The HD curves are calculated using Eq. (9) with the values of noise sources taken from Ref. 9; curves 3, 4 correspond to the total $C(n,m,t)$ and curves 5, 6 to the contribution of only velocity rates.

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