# On the spectral strength of the noise source entering the transfer impedance method

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We show that within the transfer impedance method, the spectral strength of the local noise source originally defined by Van Vliet *et al.* [J. Appl. Phys. **46**, 1804 (1975)] should be more generally replaced by the two-point spectral density of current fluctuations. The theoretical arguments are further supported by numerical results for the voltage noise spectrum associated with conduction–current fluctuations in submicron $n^+nn^+$ structures. The above two-point spectral strength is, in general, a complex quantity related to fluctuations induced by velocity fluctuations only. © *1997 American Institute of Physics.* [S0003-6951(97)01647-1]

In a seminal paper, van Vliet *et al.*[1] introduced the *transfer impedance method* (TIM) as the proper two-point internal response function, which gives a linear relation between the electric-field response and a local current perturbation, thus, enabling one to compute the internal field noise spectra originated from current fluctuations. This method, by generalizing the original impedance field method of Shockley *et al.*[2] has been widely used during the last decades for noise calculations in the framework of the drift–diffusion modeling.[2,3] The essential role of the TIM was, and actually is, to calculate the total spectrum of voltage fluctuations between the terminals of a one-port device $S_U(\omega)$. To this purpose, Ref. 1 made use of the expression

$$S_U(\omega)=\int_0^L dx\int_0^L dx'\int_0^L dx''z(x,x'',\omega)\times K(x'',\omega)z^*(x',x'',\omega), \tag{1}$$

with $K(x,\omega)$ the *spectral strength* of the noise source defined by Ref. 1 without proof as

$$K(x,\omega)\,\delta(x-x')=\overline{j_N(x,\omega)j_N^*(x',\omega)}/\Delta f, \tag{2}$$

where Eqs. (1) and (2) have been specialized to a one-dimensional geometry and, retaining the same notation of Ref. 1, $L$ is the total length of the device, $z(x,x',\omega)$ the transfer impedance matrix (i.e., the Fourier transform of the Green function of the linearized electrical response around the bias point), $j_N(x,\omega)$ the current-density noise source with the asterix denoting complex conjugate, the bar time average under ergodic conditions, and $\Delta f$ the bandwidth.

The crucial assumption of the above scheme is the *local property* in real space of the spectral strength $K(x,\omega)$, which only approximates the right-hand side of Eq. (2) corresponding to the two-point spectral density of current fluctuations.

The aim of this letter is to announce that the above assumption is unnecessary and, as such, can be replaced by the more general nonlocal definition. In the most general case of the total-current conservation equation, by assuming the linearity of fluctuations only, the voltage fluctuation between the structure terminals, $\delta U(t)$, due to a local conduction current fluctuation $\delta j_d(x',t')$ can be written as

$$\delta U(t)=-\int_0^L dx'\int_0^L dx\int_0^\infty d\tau G(x,x',\tau)\,\delta j_d(x',t-\tau), \tag{3}$$

where $G(x,x',\tau)$ is the Green function describing the electric-field response $\delta E(x,t)$ to a $\delta j_d(x',t')$ which, according to Ref. 4, is associated with the coordinate-dependent fluctuations of the distribution function with respect to its stationary value $\delta F_p(x,t)=F_p(x,t)-F_p$. By applying the Wiener–Khintchine theorem and making a time averaging, the spectral density of voltage fluctuations, $S_U(\omega)$, is given by

$$S_U(\omega)=\int_0^L dx\int_0^L dx'\int_0^L dx''\int_0^L dx'''z(x,x'',\omega)\times K(x'',x''',\omega)z^*(x',x''',\omega), \tag{4}$$

where $K(x'',x''',\omega)$ is the two-point spectral strength replacing Eq. (2) as

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$$K(x',x'',\omega)=2\int_{-\infty}^{\infty}C(x',c'',s)\exp(i\omega s)ds$$

$$=2\int_{0}^{\infty}[C(x',x'',s)\exp(i\omega s)$$

$$+C(x'',x',s)\exp(-i\omega s)]ds, \tag{5}$$

$$C(x',x'',s)=\overline{\delta j_d(x',t)\,\delta j_d(x'',t+s)^t}, \tag{6}$$

where we have taken into account the transposition property $C(x',x'',s)=C(x'',x',-s)$, which directly follows from Eq. (6). We stress once more that in deriving Eqs. (3)–(6), which generalize previous findings of Ref. 5 to a frequency-dependent noise source, the only assumption used is that of the linearity of fluctuations; any assumption concerning the conduction current fluctuations and their origin is absent.

It is evident that for spatially homogeneous systems without current flux, the cross-correlation functions of conduction current fluctuations between points $x'$ and $x''$ defined by Eq. (6) are identical for forward and reverse times, $s$ and $-s$, respectively, that is

$$C(x',x'',s)=C(x',x'',-s). \tag{7}$$

In this case, one directly obtains from Eq. (5) a well-known formula for the spectral density of current fluctuations represented through the cosinus transformation of corresponding cross-correlation functions.[1–6] In the general case, Eq. (7) is invalid and $K(x',x'',\omega)$ is described by a complex quantity whose imaginary part reflects the violation of the spatio-temporal symmetry.

We remark that Eq. (1) is recovered from Eq. (4) for $\delta(x-x')$ correlation of the current fluctuations only, i.e., when the two-point spectral strength is approximated by Eq. (2). However, the spatial correlation of the current or velocity fluctuations always exists. The physical origin of the nonlocal property of the two-point correlator is associated with the time and space operators of the Boltzmann equation, as formulated analytically in Sec. 2 of Ref. 4. Accordingly, even if the scattering process is Markovian in phase space (local and instantaneous collision limit), it is the time and space kinetics which make nonlocal the equation governing the distribution function correlator, and in turn, the correlation function of current density fluctuations. Furthermore, the existance of two-point cross-correlation of current fluctuations was analytically investigated in Refs. 4 and 6 and has been recently calculated by Monte Carlo (MC) simulations for both homogeneous[7] and inhomogeneous[8] structures. The correlation takes place within mean-free-path distances. For example, in $n$-type GaAs, this distance is of about 0.1–0.5 $\mu$m at room temperature.

Let us note that Eq. (1) remains a good approximation even in the presence of the above spatial correlation, provided the transfer impedance is a slowly varying function within the spatial correlation length (usually, this happens in long devices).

For the numerical results, we apply the nonlocal theoretical framework for the calculation of $S_U(\omega)$ to the case of a 0.3–0.6–0.4 $\mu$m $n^+nn^+$ GaAs structure with doping levels $n=5\times10^{15}$ cm$^{-3}$ and $n^+=10^{17}$ cm$^{-3}$ at $T=300$ K for a voltage of 0.6 V. Abrupt homojunctions are assumed. The

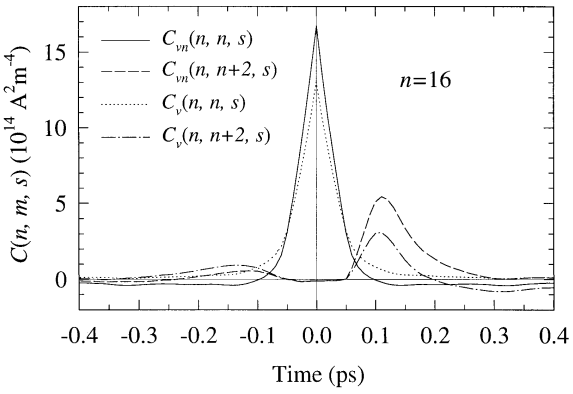


FIG. 1. Cross-correlation functions of conduction-current fluctuations in a $n^+nn^+$ GaAs structure at a voltage of 0.6 V calculated by the MC technique between the cell $n=16$ and the cells $n$ (autocorrelation) and $n+2$. The conduction-current fluctuations include number and velocity fluctuations and only velocity fluctuations.

application of the TIM is performed by means of a closed hydrodynamic (HD) approach[9] with input parameters taken from MC simulations. The function $C(x',x'',s)$ corresponding to the correlation of the local conduction current fluctuations in the structure is calculated directly by using the MC technique[9] with the frozen electric-field profile $E_s(x)$. Due to the linearization, in what concerns the coupling between fluctuations through the self-consistent electric field, two possible definitions for $\delta j_d(x,t)$ can be used to calculate $C(x',x'',s)$. A first one is to define $\delta j_d(x,t)$ as originating from both electron number and velocity fluctuations; we will denote the associated cross-correlation function as $C_{vn}(x',x'',s)$. However, since fluctuations of the electron number around a given position are induced by velocity fluctuations, a second and more plausible definition is to take $\delta j_d(x,t)$ as originating only from velocity fluctuations; we will denote the associated cross-correlation function as $C_v(x',x'',s)$.

In order to evaluate Eq. (6), the $n^+nn^+$ structure is divided into 26 cells of 500 Å each. The cross correlations between different cells $n$, $m$, $C_{vn}(n,m,s)$, and $C_v(n,m,s)$, are calculated and then introduced into Eq. (5) to obtain the two-point correlator. An example of such cross-correlation functions, calculated by means of the MC technique, is given in Fig. 1. The correlations correspond to the cell $n=16$ (in the active region near the anode) with the cells $n$ (autocorrelation) and $n+2$. While in the case of the autocorrelation function $C(n,n,s)$, the time-reversal simmetry is preserved, in the cross-correlation $C(n,n+2,s)$, this simmetry is clearly broken, the correlation being more pronounced for positive times in accord with the forward displacement of the carriers. The cross-correlation functions peak at the average time spent by carriers to cover the distance between cells without undergoing any scattering mechanism. Important differences can be observed whether number and velocity fluctuations are considered in $\delta j_d(x,t)$ or only velocity fluctuations are taken into account. The time dependence of the correlations is quite similar for both cases. However, in the former case, the correlation is practically negligible for negative times as compared with positive times. On the contrary, in the latter case, the correlation is always important and

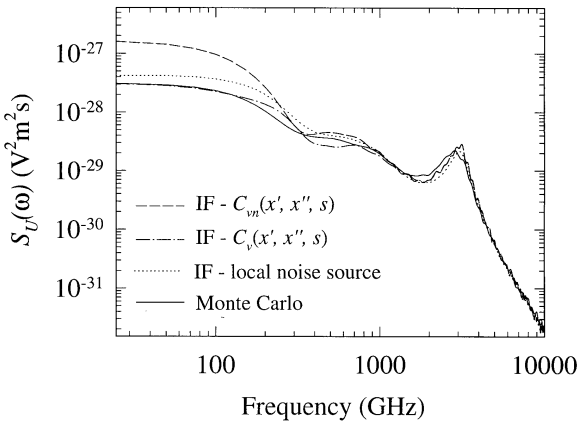


FIG. 2. Frequency dependence of the spectral density of voltage fluctuations of the structure in Fig. 1 calculated by using different techniques: MC simulation (—), IF method neglecting spatial correlations (·····), IF method including spatial correlations and considering a noise source related to number and velocity fluctuations (---) and only to velocity fluctuations (-·-).

extends over a longer time. For positive times $C_{vn}(n,m,s)$, while keeping a similar shape, takes much higher values than $C_v(n,m,s)$, which even shows a negative part. We expect that, when compared with $C_v(x',x'',s)$, $C_{vn}(x',x'',s)$ overestimates the low-frequency value of the two-point correlator.

Figure 2 shows the spectral density of voltage fluctuations between the terminals of the structure $S_U(f)$ calculated in several ways: (i) by using the TIM, including spatial correlations according to Eq. (4) and (a) using $C_{vn}(x',x'',s)$, or (b) $C_v(x',x'',s)$ to calculate the noise source; (ii) by using the TIM neglecting spatial correlations; and (iii) by using the MC technique. By naturally including all the processes responsible for the fluctuations and their correlations, the MC technique is used to validate the calculations performed with the TIM and different noise sources. It can be observed that an excellent agreement with the MC results is obtained when spatial correlations are included in the TIM calculation and only velocity fluctuations are accounted for. This agreement validates the formulation of the TIM, including spatial correlations presented here, and supports the conjecture that the most appropriate noise source is that related only to velocity fluctuations. When also number fluctuations are included in the correlator, the expected relevant overestimation of the noise, over a factor of 5, is observed. When spatial correlations are neglected and a local source of noise is considered, the results of the corresponding TIM are found to significantly overestimate the noise for about 40%. Finally, we remark that analogous calculations performed in the case of long diodes (of about 10 $\mu$m length), as expected, do not require the spatial nonlocality of the two-point correlator, and full agreement between HD and MC approaches is obtained.

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[1] K. M. Van Vliet, A. Friedman, R. J. J. Zijlstra, A. Gisolf, and A. Van der Ziel, J. Appl. Phys. **46**, 1804 (1975); **46**, 1814 (1975).
[2] W. Shockley, J. A. Copeland, and R. P. James, in *Quantum Theory of Atoms, Molecules and Solid State*, edited by P. O. Lowdin (Academic, New York, 1966), p. 537.
[3] G. Ghione and F. Filicori, IEEE Trans. Comput.-Aided Des. **12**, 425 (1993).
[4] S. Gantsevich, V. Gurevich, and R. Katilius, Riv. Nuovo Cimento **2**, (1979).
[5] K. M. van Vliet and H. Mehta, Phys. Status Solidi B **106**, 11 (1991).
[6] J. P. Nougier, J. C. Vaissiere, and C. Gontrand, Phys. Rev. Lett. **51**, 513 (1983).
[7] J. Mateos, T. González, and D. Pardo, J. Appl. Phys. **77**, 1564 (1995).
[8] J. Mateos, T. González, and D. Pardo, Appl. Phys. Lett. **67**, 685 (1995).
[9] P. Shiktorov, V. Gružinskis, E. Starikov, L. Reggiani, and L. Varani, Appl. Phys. Lett. **68**, 1516 (1996).