Acceleration fluctuation scheme for diffusion noise sources within a generalized impedance field method

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(Received 20 November 1997)

It is shown that the spectral density of the diffusion noise source can be described in terms of accelerations caused by single scattering events. Within a hydrodynamic approach a response-function formalism for the spectral density of voltage fluctuations in semiconductor structures is developed on the basis of a generalized impedance field method. The formalism is validated by comparison with Monte Carlo simulations of *n*-type GaAs structures. [S0163-1829(98)01316-2]

In its most advanced form, the impedance field (IF) method expresses the spectral density of voltage fluctuations between the terminals of a one-port device $S_U(\omega)$ in terms of spatial convolution between the local IF $\nabla Z(x,\omega)$ and the two-point spectral density of the current-density fluctuations $S_{jj}(x,x',\omega)$ treated as the macroscopic source of fluctuations:[1–5]

$$S_U(\omega)=\int_0^L dx\int_0^L dx'\,\nabla Z(x,\omega)S_{jj}(x,x',\omega)\nabla Z^*(x',\omega), \tag{1}$$

where the above equation is specialized to a one-dimensional geometry with L being the length of the device. The main drawback of such an approach is that the spectral density of current fluctuations does not represent the proper microscopic noise source since it already contains the dynamic contribution of carriers while moving between scattering events. This complicates noise calculations of current submicrometer devices where the short time and space scales require one to account for the spatiotemporal correlation of the current-density fluctuations, which can only be evaluated by means of the direct Monte Carlo (MC) simulation of the device being tested.[5,6] In contrast, for the Markovian processes considered here, the microscopic noise sources are the scattering events themselves, which are uncorrelated in both space and time. A formal expression for $S_U(\omega)$ able to include the proper Markovian noise sources and to attribute the full dynamics to an equivalent IF rigorously derived within a hydrodynamic (HD) theory would overcome the above drawback.

The aim of this paper is to address this issue. For this sake let us consider first the nature of fluctuation sources. In a microscopic description, the electron motion along the field direction consists of a stochastic sequence of accelerations $a(t)$. The source of accelerations is twofold: One comes from the electric field E and the other from the stochastic impulsive forces associated with single scattering events. These impulsive forces are proportional to $\Delta v_i\delta(t-t_i)$ and are associated with instantaneous changes of the electron velocity at a given time moment $t=t_i$ by a value of $\Delta v_i=v_{ai}-v_{bi}$, v_{ai} and v_{bi} indicating the electron velocity just after and before the ith scattering event. Under stationary conditions in the bulk, the average value of $a(t)$ is equal to zero for both the ensemble and time averaging along a sufficiently long trajectory of a single electron due to the ergodicity of the process. This allows us to consider the time dependence of the sequence of accelerations of a single electron during the time interval $0\leqslant t\leqslant T$ given by

$$a(t)=eEm^{-1}(t)+\sum_{i=1}^{N}\Delta v_i\delta(t-t_i), \tag{2}$$

where N is the number of scattering events during T and $m(t)$ is the electron effective mass that can depend on time due to nonparabolicity, intervalley transitions, etc. The electric field E is considered to be constant in time. In the general case the autocorrelation function of electron accelerations $C_a(s)$, which in accordance with Eq. (2) determines the random walk of a carrier, can be decomposed into two parts as

$$C_a(s)=\overline{a(t)a(t+s)}^t=C_a^S(s)+C_a^L(s). \tag{3}$$

The first part

$$C_a^S(s)=\frac{1}{T}\sum_{i=1}^{N}(\Delta v_i)^2\delta(s) \tag{4}$$

is the *short-time correlation* of the system and is given by the autocorrelation of acceleration fluctuations at the time moments of scattering events. Let us stress that $C_a^S(s)$ is proportional to a $\delta(s)$ function, thus reflecting the Markovian nature of the scattering process.

The second part

$$C_a^L(s)=\frac{1}{T}\sum_{i\neq j}^{N}\Delta v_i\Delta v_j\delta(s-(t_i-t_j)) + \frac{eE}{T}\sum_{i=1}^{N}\Delta v_i[m^{-1}(t_i-s)+m^{-1}(t_i+s)] + \frac{(eE)^2}{T}\int_0^T m^{-1}(t)m^{-1}(t+s)dt \quad (5)$$

is the *long-time correlation* of the system and consists of three terms. The first, second, and third terms on the right-hand side of Eq. (5) correspond, respectively, to the cross correlation of accelerations caused by scattering events, the cross correlation between scattering accelerations and the external field E, and the autocorrelation of the external field E.

To illustrate the main features of $C_a(s)$ and the transition from the stochastic description to that based on the HD approach we consider the following simple model: (i) Scattering events are fully randomizing, so that $\langle v_{ai}\rangle=0$; (ii) scattering events occur with a constant rate $\gamma=1/\tau$; and (iii) the effective mass is constant in time. In the framework of this model, simple analytical expressions for $C_a(s)$ and its spectrum can be obtained. The second and third terms of the long-time correlation [see Eq. (5)] reduce to constant values respectively given by $-2(eE/m)^2$ and $(eE/m)^2$. They exactly compensate for the long-time limit $s\to\infty$ value $(eE/m)^2$ of the first term, so that the total correlation function of acceleration fluctuations takes the form

$$C_a(s)=\frac{1}{\tau}\langle\Delta v_i^2\rangle\left[\delta(s)-\frac{\exp\left(-\frac{|s|}{\tau}\right)}{2\tau}\right], \quad (6)$$

where the first and second terms in the square brackets correspond to the short- and long-time parts of the correlation function of acceleration fluctuations. As it follows from Eq. (6), despite the fact that scattering events are uncorrelated, the accelerations caused by them have a long-time correlation that compensates for any initial fluctuation, thus providing the relaxation of the system. The spectral density of accelerations is given by

$$S_a(\omega)=2\frac{\langle\Delta v_i^2\rangle}{\tau}\left[1-\frac{1}{1+(\omega t)^2}\right]. \quad (7)$$

When $\omega\to0$, the spectral density of the short-time part of $C_a(s)$ is fully compensated for by the spectral density of its long-time part, so that $S_a(\omega)\sim\omega^2$. The latter is responsible for the finite value taken by the spectral density of velocity fluctuations at $\omega\to0$:

$$S_v(\omega)=\frac{S_a(\omega)}{\omega^2}=2\langle\Delta v_i^2\rangle\frac{\tau}{1+(\omega\tau)^2}. \quad (8)$$

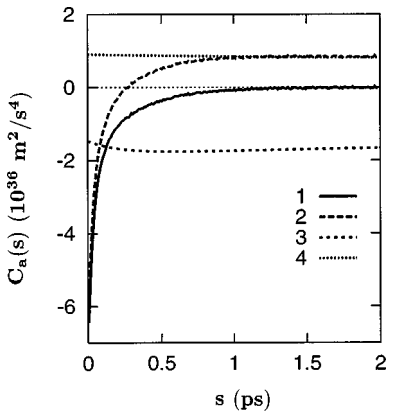

FIG. 1. Long-time part of the correlation function of acceleration fluctuations (curve 1) together with its components [curves 2, 3, and 4 corresponding to the first, second, and third terms on the right-side of Eq. (5)]. Calculations are performed with a MC simulation of bulk n-type GaAs at $E=5$ kV/cm and $T=300$ K.

At thermal equilibrium, by substituting $\langle\Delta v_i^2\rangle=2\langle v^2\rangle=2k_BT_0/m$ into Eq. (8) one obtains the standard relation for $S_v(\omega)$.[7]

As an application, we have performed MC simulations for the case of hot electrons in bulk n-type GaAs at $E=5$ kV/cm and $T=300$ K. Figures 1 and 2 illustrate the numerical results of the simulations for the various parts of the correlation function $C_a(s)$ (Fig. 1) and of the associated spectral density $S_a(f)$ (Fig. 2). Figure 1 reports the three components (curves 2–4, respectively) of the long-time part of $C_a(s)$ and their sum (curve 1). The short-time part of $C_a(s)$, being proportional to a δ function, is omitted in Fig. 1, but the corresponding spectral density, which is independent of frequency, is reported in Fig. 2 by curve 2 together with the long-time correlated part (curve 3) and their sum (curve 1). It can be observed that the results of the numerical calculations are in full agreement with what is predicted by the simple model.

The representation of the spectral density of velocity fluctuations in terms of Eq. (8) is identical to the description of

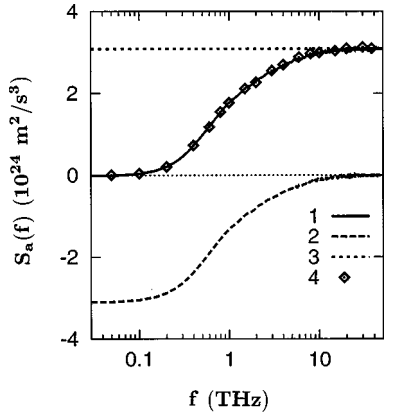

FIG. 2. Spectral density of the correlation functions in Fig. 1. Curve 1 refers to the total spectrum and curves 2 and 3 refer to its short- and long-time correlated parts, respectively. Curve 4 refers to the spectral density of acceleration fluctuations calculated as $S_a(\omega)=\omega^2S_v(\omega)$ from the spectral density of velocity fluctuations obtained by MC simulation.

fluctuations in the framework of the Langevin approach for velocity fluctuations described by[7]

$$\frac{d}{dt}\,\delta v = -\gamma \delta v + f(t), \tag{9}$$

where the Langevin random force $f(t)$ is characterized by the white spectral density $S_f = 2\langle \Delta v_i^2 \rangle / \tau$. The random force is determined by the short-time part of $C_a(s)$, thus corresponding to the autocorrelation of accelerations at single scattering events. The relaxation term $\gamma \delta v$ is determined by the long-time part of $C_a(s)$, thus corresponding to the cross correlation of successive accelerations. As a consequence, only the short-time part of $S_a(\omega)$ corresponds to the Langevin source of uncorrelated fluctuations and as such it must be treated as the source of noise at a HD level. In the general case of a HD approach when balance equations for different moments (e.g., velocity and energy) are considered the random force associated with each equation is determined by the short-time part of the correlation function of the considered moment acceleration during scattering events. Accordingly, the long-time part corresponds to the relaxation terms already included in the respective equations. The main advantage of such an approach is that the noise sources now are, by definition uncorrelated in both time and space since they are connected with fluctuations induced by single scattering events only.

By considering the accelerations in momentum and energy space as the microscopic source of fluctuations, let us restrict ourselves to a HD approach based on the drift velocity and mean energy conservation equations[8,9]

$$\dot{v} = eEm^{-} - v\,\nu_v - v\,\frac{\partial v}{\partial x} - \frac{1}{n}\,\frac{\partial}{\partial x}\,(nQ_v) + \xi_v(t), \tag{10}$$

$$\dot{\varepsilon} = eEv - (\varepsilon - \varepsilon_{th})\,\nu_\varepsilon - v\,\frac{\partial \varepsilon}{\partial x} - \frac{1}{n}\,\frac{\partial}{\partial x}\,(nQ_\varepsilon) + \xi_\varepsilon(t), \tag{11}$$

with the usual meaning of symbols and where the sources of fluctuations are the random forces $\xi_v(t)$ and $\xi_\varepsilon(t)$. In accordance with the general approach developed in Refs. 5 and 8, the voltage fluctuations between the structure terminals due to the velocity and energy fluctuations can be represented as

$$\delta U(t) = \int_0^L dx \int_0^\infty d\tau [R_v(x,\tau) n(x)\, \delta v(x, t-\tau) + R_\varepsilon(x,\tau) n(x)\, \delta\varepsilon(x, t-\tau)], \tag{12}$$

where R_v and R_ε are the response functions of the voltage perturbation caused by spontaneous fluctuations of velocity δv and energy $\delta\varepsilon$ at point x due to scatterings. By applying to the $\delta U(t)$ given by Eq. (12) the Wiener-Khintchine theorem, the spectral density and voltage fluctuations can be expressed in a form similar to the IF formula of Eq. (1):

$$S_U(\omega) = \sum_{\alpha,\beta} \int_0^L \nabla Z_\alpha(x,\omega) \nabla Z_\beta^*(x,\omega) n(x) S_{\dot{\alpha}\dot{\beta}}\, dx, \tag{13}$$

where α and β stand for v and ε, $\nabla Z_{\alpha,\beta}$ is the *generalized IF* associated with perturbation of α and β, and

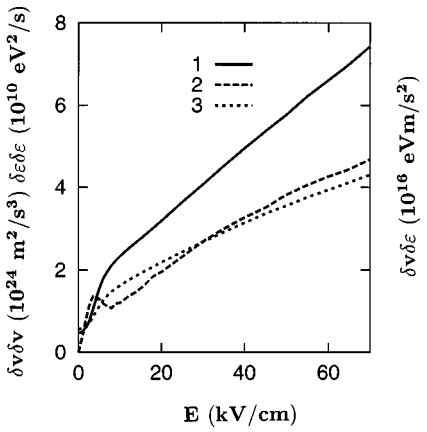

FIG. 3. Field dependence of the spectral density of the microscopic noise sources $S_{\dot{v}\dot{v}}$, $S_{\dot{v}\dot{\varepsilon}}$, and $S_{\dot{\varepsilon}\dot{\varepsilon}}$ (curves 1, 2, and 3, respectively). Note the frequency independence of the above spectra.

$$S_{\dot{\alpha}\dot{\beta}} = \frac{2}{T} \sum_{i=1}^{N} \Delta\alpha_i \Delta\beta_i \tag{14}$$

are the corresponding noise sources with a white spectrum. Here $\Delta\alpha_i, \Delta\beta_i$ correspond to instantaneous variations of velocity and energy during the ith scattering event. Equations (13) and (14) are the main result of the present theory and satisfy the requirement of describing voltage fluctuations with the proper Markovian microscopic noise sources and the associated generalized IF's.

Together with all parameters of the model, namely, $m^{-1}, \nu_v, \nu_\varepsilon, Q_v, Q_\varepsilon$, the noise sources are assumed to depend only on the local mean energy and as such can be obtained from stationary MC simulations of the bulk material at the given constant electric field. The field dependence of the noise sources calculated with the MC method for bulk n-type GaAs is presented in Fig. 3. Its systematic increase for fields above about 5 kV/cm is due to the presence of hot-carrier effects.

Figure 4 reports the results of noise calculations obtained for a submicrometer (0.3–0.6–0.4)-μm n^+nn^+-type GaAs structure with doping levels $n = 5\times10^{15}$ cm^{-3} and $n^+ = 10^{17}$ cm^{-3} at $T = 300$ K and an applied voltage of 0.6 V.

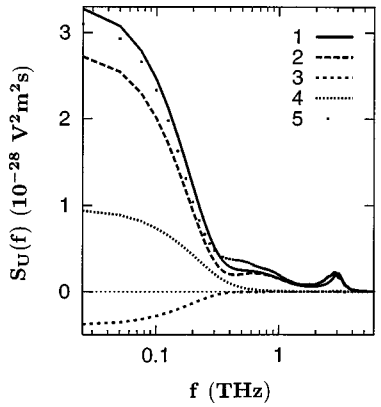

FIG. 4. Spectral density of voltage fluctuations, $S_U(f)$ calculated by Eq. (13) and its three microscopic contributions corresponding to $S_{\dot{v}\dot{v}}$, $S_{\dot{v}\dot{\varepsilon}}$, and $S_{\dot{\varepsilon}\dot{\varepsilon}}$ (curves 1, 2, 3, and 4, respectively) together with $S_U(f)$ obtained directly by MC simulation of a submicrometer n^+nn^+ n-type GaAs diode at $U = 0.6$ V (curve 5).

Abrupt homojunctions are assumed. The calculation of the generalized IF (and the associated Green's functions) for velocity and energy perturbations is performed by means of the HD approach[8] with input parameters taken from MC simulations. The excellent agreement for the $S_U(\omega)$ obtained within the present approach (solid curve) and by the direct MC simulation of the voltage noise (dots) fully supports the physical reliability of the microscopic theory here developed.

In conclusion, the theoretical scheme we propose for the spectral density of voltage fluctuations shows that the spectral density of the microscopic fluctuation source can be described in terms of accelerations caused by single scattering events. As a result, a generalization of the IF method is developed in the framework of the velocity and energy conservation equations when the accelerations in momentum and energy space during scattering event are used as sources for fluctuations.

This work has been performed within the Italian-Lithuanian Project ''Research and Development Cooperation in Submicron Electronics'' and supported by the Italian Minister of Foreign Affairs. Partial support from the NATO networking linkage Grant No. HTECH.LG 960931, Computer Networking Supplement No. CNS 970627, MADESS II Project of the Italian Consiglio Nazionale delle Ricerche, and Project No. SA 11/96 from the Consejería de Educación y Cultura de la Junta de Castilla y León is gratefully acknowledged.

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